Math 322

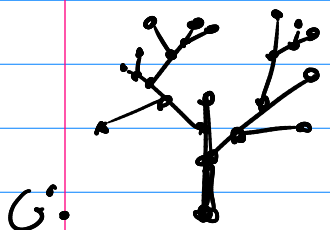
Due Monday

① construct a decision tree for finding a possible fake coin among 6 coins.
- What is best possible number of weighings to find the possible fake (or state it isn't there)?
- Can you construct such a tree?

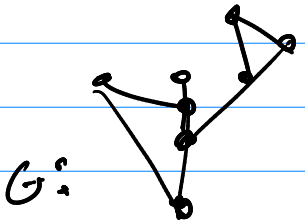
10.4(1,2,3) (expression trees)

Trees

Def: Undirected Graph with no simple circuits is a tree.



G: tree

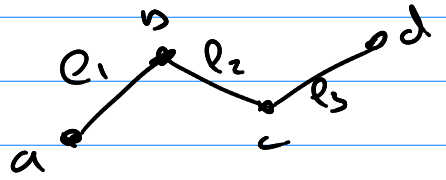


G: not a tree

Th^m (these are all logically equiv)

① $G = (V, E)$ is a tree (no simple circuits)

② between every distinct pair of vertices there exists a unique simple path between them.



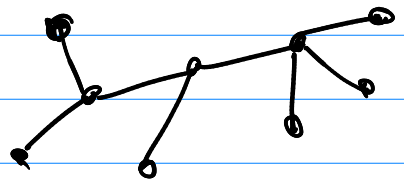
unique path e_1, e_2, e_3 for a to d

③ a graph $G=(V,E)$ is connected and $\forall e \in E$

$G' = (V, E - \{e\})$, then G' is disconnected.

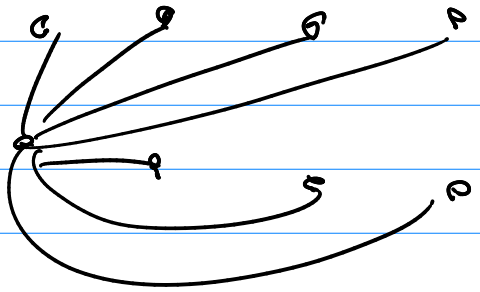
(all $e \in E$ are cut edges)

④ a graph contains no simple circuits and adding one edge will create a simple circuit.



⑤ a graph G is (connected) and $|E| = |V| - 1$

⑥

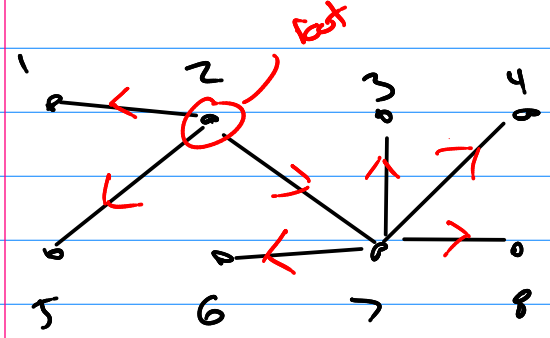


Terms: ① G is tree if it has no simple circuits (see above)

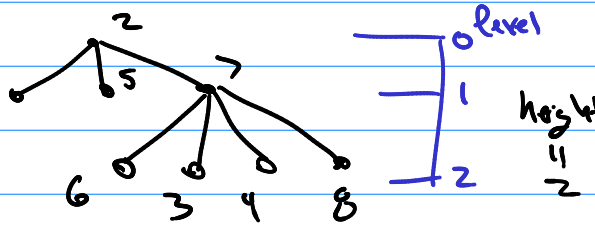
② Rooted tree a) pick a vertex and call it the root

b) make G directed by all directions go from the root

c) re-draw with direction = down



(rooted tree)

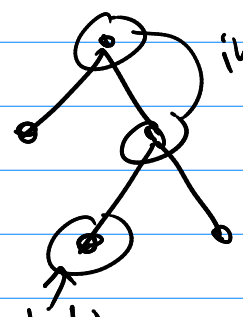


terms:

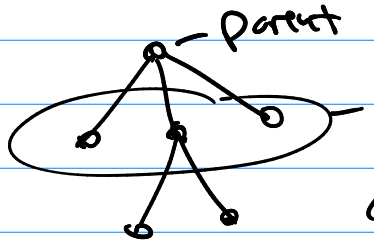
① level = length of path back to root

② height = longest level

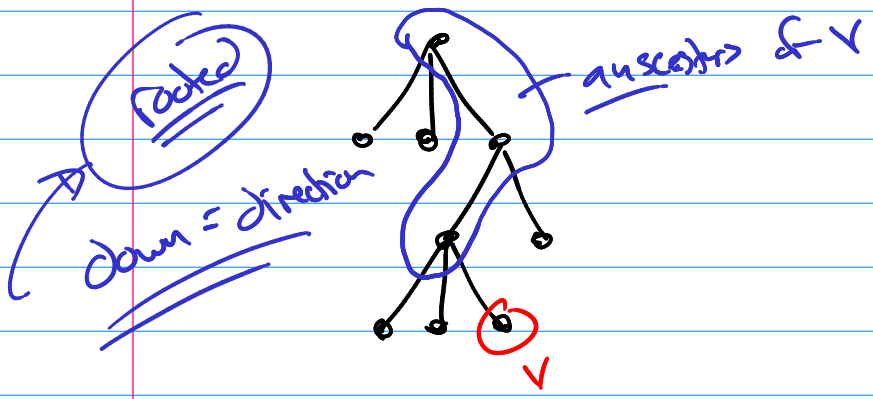
③

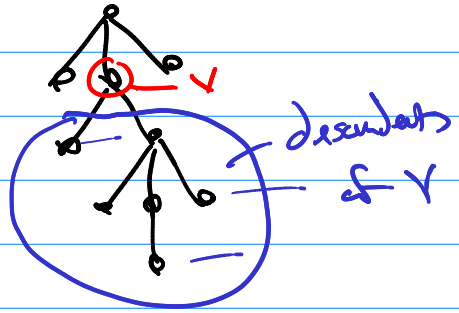


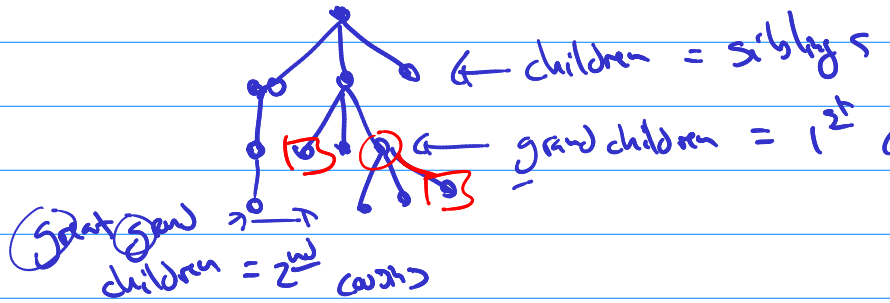
internal vertices or parents



no children = leaf







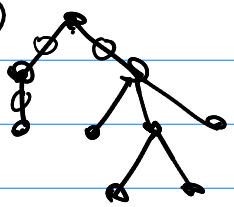
④ m-ary tree (rooted tree)

for all internal vertices if

m = largest number of children

→ G is an m-ary tree

Ex



⇝ 3-ary tree

(5) full n-ary : all internal vertices have exactly n children.

Properties (1) leaf and internal vertex are disjoint

$$\ell + i = |V| = n$$

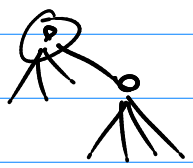
(2) $|\text{children}| = n - 1$

(3) full n-ary $|\text{children}| = ni$

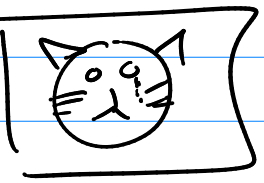
(4) $n = ni + 1$

So $n = i + \ell$, $n = ni + 1$ } full n-ary tree

(Ex) Chain text: Send to 10 people or the cute cat dies!



full 10-ary.

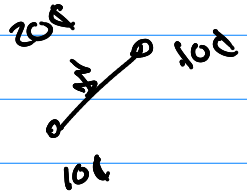


$n = i + \ell$, $n = 10i + 1$ given $i = 1000$ (sent text)

$n = ?$ $\ell = ?$

$n = 1000 + \ell$

$n = 10(1000) + 1 = 10{,}001 \rightarrow \ell = 9{,}001$

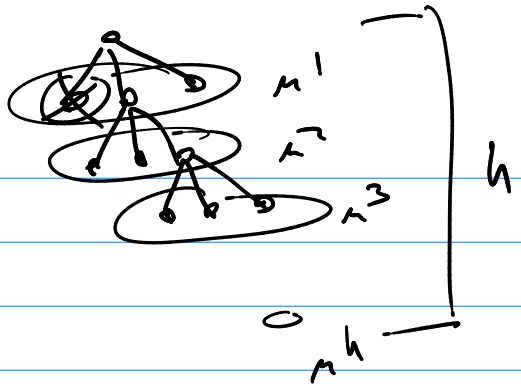


$|E| = 10{,}000$ (sent/rec text) costs 20¢

$2000

,3000.

Th^n $l \leq m^h$



$\rightarrow \log_m l \leq \log_m m^h$

$h \geq \lceil \log_m l \rceil$

back to text example

$n = i + l$

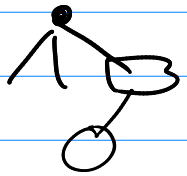
$n = 10i + 1$

(not told n, i, l)

text

Fwd : Fwd : Fwd

→ blah blah



n $h = 4$ $\quad l \leq m^h$ so $l \leq 10^4 = 10{,}000$

So let $l = 10{,}000$

$n = i + 10000$

$n = 10i + 1$

Solve

Apps ① Spanning / Minimal Spanning trees

given $G = (V, E)$ that is connected

create Tree $T = (V, E')$ from G

by removing edges.

(c) Decision Trees

internal vertex = decision
child = outcome from decision
leaf = end results of a process of decisions.

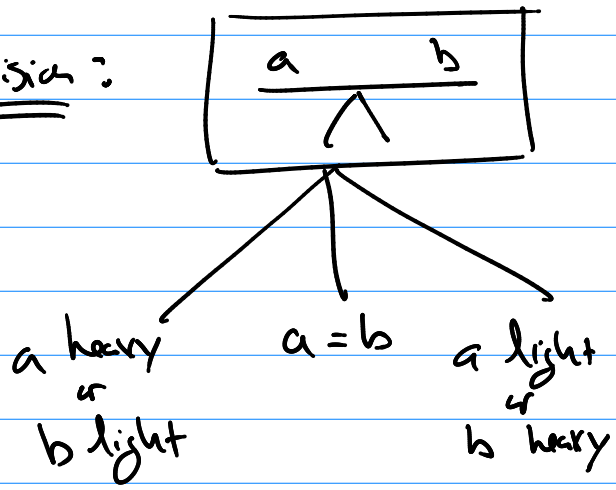
(ex) find a fake coin that is light or heavy among 4 coins

c_1, c_2, c_3, c_4

leaves: $1^h, 1^l$ (heavy, light)
$2h, 2l$
$3h, 3l$
$4h, 4l$

$|\text{leaves}| = 8$

decision:



best case: $h \geq \lceil \log_n l \rceil = \lceil \log_3 8 \rceil = 2$

outcomes: $1h, 1l, 2h, 2l$
$3h, 3l, 4h, 4l$