Math 511

$\boxed{\text{Q's}}$ Solving systems of linear equations

terms:

- Square $n \times n$ (eqn's, unknowns)
- non-square $m \times n$
  - $m > n$ over determined
  - $m < n$ under determined
- $\boxed{\text{Homogeneous}}$

$$\begin{cases} \text{expression } \#1 = 0 \\ \text{expression } \#2 = 0 \\ \vdots \\ \text{expression } \#m = 0 \end{cases}$$

a) always has $x_1 = 0,\ x_2 = 0, \dots, x_n = 0$ trivial soln

b) may have ∞ solns (non-trivial solns)

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ex

$$\begin{cases} 3x_1 + x_2 - x_3 = 0 \\ x_1 - x_2 + 2x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 0 \\ 1 & -1 & 2 & 0 \end{array}\right] \quad \left(\begin{array}{l} r_1 \text{ swap } r_2 \\ \text{then} \\ -3r_1 + r_2 = New\ r_2 \end{array} \right)$$

lead coef (1st non zero)

$$\left[\begin{array}{ccc|c} \boxed{1} & -1 & 2 & 0 \\ 0 & \boxed{4} & -7 & 0 \end{array}\right] \leftarrow 4x_2 - 7x_3 = 0$$

lead coef

$x_1 \quad x_2 \quad x_3$ no lead $\rightarrow$ x_3 is free to be $\boxed{\text{any}}$ number!

say $x_3 = \alpha$

$x_2 = \frac{7}{4}\alpha$ and $x_1 - x_2 + 2x_3 = 0$

$x_1 - \underbrace{\frac{7}{4}\alpha + 2\alpha}$ $= 0$

$x_1 + \frac{1}{4}\alpha = 0$

$x_1 = -\frac{1}{4}\alpha$

$$\boxed{\text{Soln} \Rightarrow \left(-\frac{1}{4}\alpha, \frac{7}{4}\alpha, \alpha\right)}$$

Matrices - rectangular block of numbers

ex
$$\begin{bmatrix} 0 & 2 & 1 \\ 1 & 3 & -1 \end{bmatrix} \quad 2 \times 3$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 2 & 1 \end{bmatrix} \quad 3 \times 3$$

"Variable" Matrix

$$\underset{m \times n}{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]$$

Special Matrices

use bold font

1. **Vectors**
   - row vector $1 \times n$ size $\vec{v}$
   - column vector $m \times 1$ size $\mathbf{v}$
2. $\underset{n \times n}{I} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$ ex $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or . .
3. $\underset{m \times n}{0} = [0]$ (all zero matrix) $a_{ij} = 0$

Math

= toys + rules　　Now　　toys = Matrices / Vectors

Rules?

① Equality　　$A = B$　　① both $m \times n$

② $a_{ij} = b_{ij}$ for all i, j

② Addition　　$A + B = [a_{ij} + b_{ij}]$ but both $m \times n$

back to sys. of linear eqn's

$$(2x + y) + (-x + 3y) = x + 4y$$

$$\underset{a_{11}}{[2 \quad 1]} + \underset{b_{11}}{[-1 \quad 3]} \qquad \underset{a_{11}+b_{11}}{[1 \quad 4]}$$

③ Scalar multiplication

$$\alpha A = [\alpha a_{ij}] \quad \text{ex}\quad 3\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = ?$$

sys. of linear eqn's　　$3(x + 2y) = 3x + 6y$

$$3[1 \quad 2] = [3\cdot 1 \quad 3\cdot 2]$$

Sub: $A - B = A + (-1)B = [a_{ij}] + [-b_{ij}]$

$$= [a_{ij} - b_{ij}]$$

Multiplication?　　Matrix $\underbrace{\text{(times?)}}_{\text{Matrix multiplication}}$ Matrix

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 2 \\ -2 & 3 & 9 \end{bmatrix}$$

2×2 $\quad 2 \times 3$ $\quad 2 \times 3$

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to build to $AB = ?$ need ...

Definition ① (row vector)(col vector) = scalar

$$\vec{v} \vec{w} = c = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$

ex $$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1\cdot 1 + 2\cdot 2 + 3\cdot 3 = 14$$

from $$2x - y + z = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$