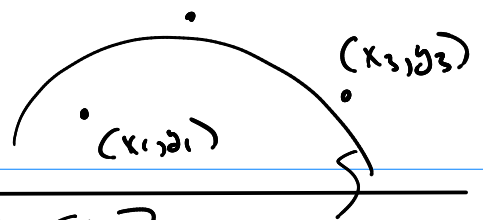


Math 511



Q5

z.z #12

$$\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$y = a + bx + cx^2$$

$$y_1 = a + bx_1 + cx_1^2$$

$$y_2 = a + bx_2 + cx_2^2$$

$$y_3 = a + bx_3 + cx_3^2$$

a)  $\det(V)$

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 \\ 0 & x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix}$$

type 3 row op

$$= \begin{vmatrix} x_2 - x_1 & x_2^2 - x_1^2 \\ x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix}$$

$$= (x_2 - x_1)(x_3^2 - x_1^2) - (x_3 - x_1)(x_2^2 - x_1^2)$$

$$= (x_2 - x_1)(x_3 - x_1) \left[ (x_3 + x_1) - (x_2 + x_1) \right]$$

b) (non-singular)  $\equiv (\det(V) \neq 0) \equiv (V \text{ is invertible})$   
 $\equiv (V^{-1} \text{ exists})$

(singular)  $\equiv (\det(V) = 0) \equiv (V \text{ is not invertible})$   
 $\equiv (V^{-1} \text{ does not exist})$

Singular

$$\det(V) = 0$$

$$(x_2 - x_1)(x_3 - x_1) / (x_3 - x_2) = 0$$

$$x_2 = x_1 \text{ or } x_3 = x_1 \text{ or } x_3 = x_2$$

non-singular

$$\det(V) \neq 0$$

$$x_2 \neq x_1 \text{ and } x_3 \neq x_1 \text{ and } x_3 \neq x_2$$

(24)  $\det \begin{pmatrix} 1 & 2 & 0 & 1 \\ 3 & -1 & 0 & 2 \\ 1 & 3 & 1 & 2 \\ 0 & 1 & 2 & 2 \end{pmatrix}$  by refactor =  $+0 - 0 + (1) \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$

=  $\begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 3 & 2 \end{vmatrix}$

=  $+0 - 1 \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 3 & 2 \end{vmatrix}$  elim. (type 3)

=  $-13 + \begin{vmatrix} 1 & 2 & 1 \\ 0 & -7 & -1 \\ 0 & 1 & 1 \end{vmatrix}$  type 1

=  $-13 + \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -7 & -1 \end{vmatrix}$

=  $-13 - \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = -13 - 0 = -13$

Note:  $\begin{bmatrix} 1 & 2 & -1 & | & 1 & 0 & 0 \\ 3 & 1 & 2 & | & 0 & 1 & 0 \\ 1/2 & 1 & 0 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & 1/2 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -1 & | & 1 & 0 & 0 \\ 0 & -5 & 5 & | & -3 & 1 & 0 \\ 0 & 0 & 1/2 & | & -1/2 & 0 & 1 \end{bmatrix}$

We could continue to find  $A^{-1}$   
 $\rightarrow$   $[I | A^{-1}]$

but we can push to see

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = U = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Exam 1 11 probs @ 10pts  
100 pts = 100%

1.1 (1 prob) → system of eqn's solve by Sub. or elimin

1.2 (2 probs) aug. matrix slvr system of eqn's

1.3 (2 probs) Matrix arithmetic

look ops  $A+B, AB, \alpha A, A^2, A^3, \text{ etc}$

1.4 (1 prob) Matrix Algebra (assume inverse exists)  
(or given it)

(\*) find

$U$

$$UA - U + B^2 = 3U + C$$

$$UA - UI - 3UI = C - B^2$$

$$UA - UI - U(3I) = C - B^T$$

etc

1.5 (3 parts)

① Find  $A^{-1}$

② For some  $A$  as } find  $A = LU$

③ - State th<sup>n</sup> 1.3.1 give example of it

- State th<sup>n</sup> 1.5.2 give example of it

from me → ③ a prob. that uses understanding the th<sup>n</sup>'s

1.6 (0 parts)

(2 parts)

2.1  
2.2

}  $\det(A)$

and explain how they relate to  $A^{-1}$ , singular, etc.