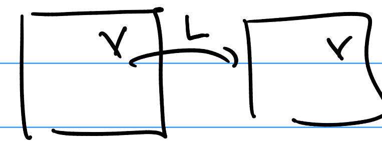
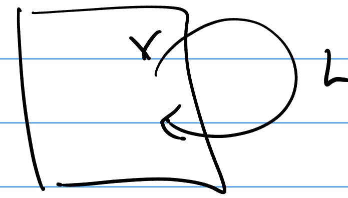
Math 511

$\boxed{\text{Q's}}$ 4.1 #17 $\boxed{\text{term}}$ $L: V \to V$



use #1 $\boxed{L \text{ on } V.}$



#2 L is a linear transformation mapping L on V? Use the name Linear operator

17a) $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_E\right) = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}_E$

(not part of problem $\boxed{\text{but}}$ is it a Linear Operator?)

2-step check ① $L(\alpha \mathbf{x}) = \alpha L(\mathbf{x})$

② $L(\mathbf{x}_1 + \mathbf{x}_2) = L(\mathbf{x}_1) + L(\mathbf{x}_2)$

① $L(\alpha \mathbf{x}) \stackrel{?}{=} \alpha L(\mathbf{x})$

$$L\left(\alpha \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) \stackrel{?}{=} \alpha\, L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$$

$$L\left(\begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix}\right) \stackrel{?}{=} \alpha \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha x_3 \\ \alpha x_2 \\ \alpha x_1 \end{bmatrix} \stackrel{?}{=} \alpha \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}$$ true

② $L(\mathbf{x}_1 + \mathbf{x}_2) \stackrel{?}{=} L(\mathbf{x}_1) + L(\mathbf{x}_2)$

$$L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right) \stackrel{?}{=} L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) + L\left(\begin{bmatrix} d \\ e \\ f \end{bmatrix}\right)$$

$$L\left(\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}\right) \stackrel{?}{=} \begin{bmatrix} c \\ b \\ a \end{bmatrix} + \begin{bmatrix} f \\ e \\ d \end{bmatrix}$$

$$\begin{bmatrix} c+f \\ b+e \\ a+d \end{bmatrix} \overset{?}{=} \begin{bmatrix} c \\ b \\ a \end{bmatrix} + \begin{bmatrix} f \\ e \\ d \end{bmatrix}$$ true

So $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}$ is a linear operator

$$\ker(L) = \{ \mathbf{x} \mid \boxed{L(\mathbf{x}) = \mathbf{0}} \}$$ who goes to $\mathbf{0}$?

$$\begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So $\ker(L) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$$\text{range}(L) = L(\mathbb{R}^3) = \{ \mathbf{y} \mid \text{there is an } \mathbf{x} \ L(\mathbf{x}) = \mathbf{y} \}$$

any $\mathbf{x}$ in $\mathbb{R}^3$ → ex $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

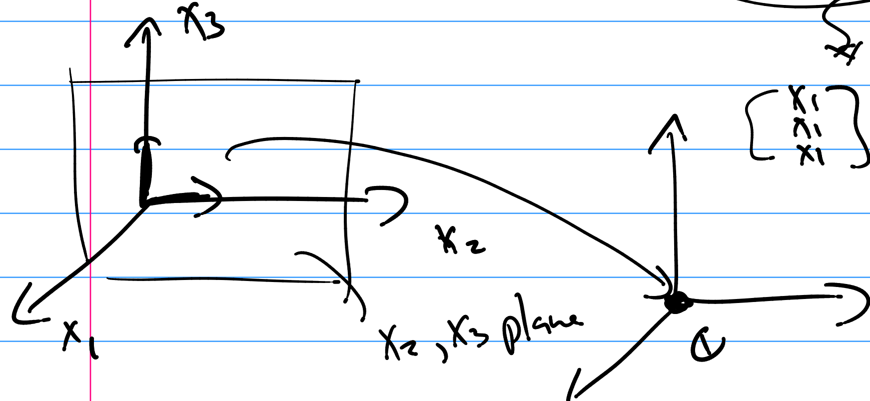
$$L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \boxed{\begin{bmatrix} c \\ b \\ a \end{bmatrix}} \to \text{all } \mathbb{R}^3 \qquad \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

17c) $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix}$

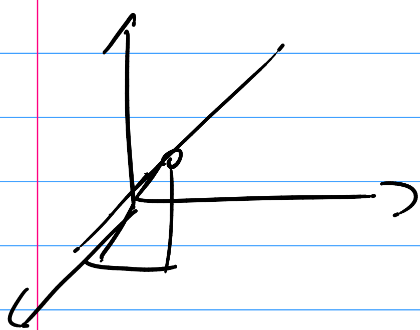
$$\ker(L) = \{ \mathbf{x} \mid L(\mathbf{x}) = \mathbf{0} \} = x_2, x_3 \text{ plane}$$

$\text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$

$$\begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ → $x_1 = 0$, $x_2 =$ anything, $x_3 =$ anything.



$\text{range}(L) = L(\mathbb{R}^3) = \{ \vec{y} \mid \text{there is an } \vec{x},\ L(\vec{x}) = \vec{y} \}$



$L(\vec{x}) = \begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\text{range}(L) = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)$

11.2 Can we find a matrix to represent L?

row swap of 1, 3?

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

17a) $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_E \right) = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}_E$

standard matrix representing L: $A[\vec{x}]_E = [\vec{y}]_E$

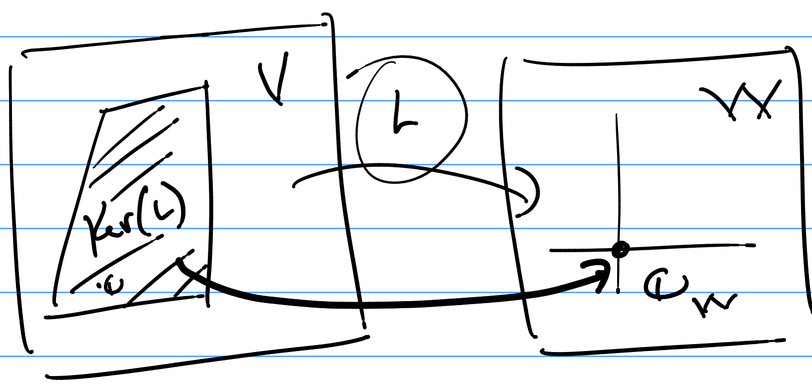
same as $L([\vec{x}]_E) = [\vec{y}]_E$

4.2 says $A = \left[L\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ L\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ L\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$

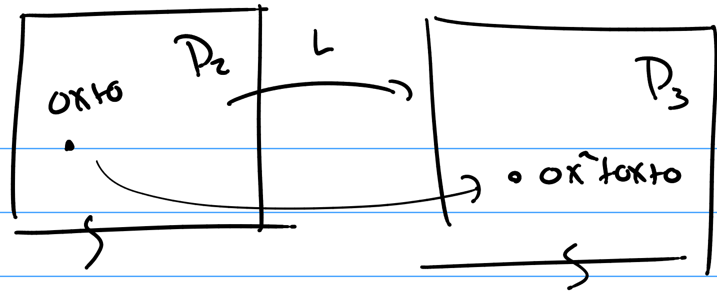
$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

4.1 Kernel (just rethink what null space was)

is a kernel



ex



$p = ax+b$ $\qquad$ $p = ax^2+bx+c$

$$\text{Anti} : P_2 \to P_3 \qquad \text{Anti}\,(ax+b) = \tfrac{1}{2}ax^2 + bx$$

or $\qquad \text{Anti}\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2}a \\ b \\ 0 \end{bmatrix}$

① is Anti( ) a $\underline{\underline{\text{linear trans. mapping}}}$?

$\underline{\underline{\text{2-step check}}}$ $\quad$ ① $L(\alpha X) \overset{?}{=} \alpha L(X)$

$$\begin{bmatrix} \frac{1}{2}a\alpha \\ b\alpha \\ 0 \end{bmatrix} \overset{?}{=} \alpha \begin{bmatrix} \frac{1}{2}a \\ b \\ 0 \end{bmatrix} \quad \underline{\underline{\text{true}}}$$

② $L(X_1 + X_2) \overset{?}{=} L(X_1) + L(X_2) \qquad X_1 = \begin{bmatrix} a \\ b \end{bmatrix} \quad X_2 = \begin{bmatrix} c \\ d \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{2}(a+c) \\ b+d \\ 0 \end{bmatrix} \overset{?}{=} \begin{bmatrix} \frac{1}{2}a \\ b \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}c \\ d \\ 0 \end{bmatrix} \quad \underline{\underline{\text{true}}}$$

So Anti( ) $\boxed{\text{is}}$ a linear trans.

② $\text{Ker}(\text{Anti}) = ?$ $\qquad \text{Ker}(\text{Anti}) = \{ p \mid \text{Anti}(p) = 0 \}$

$$\tfrac{1}{2}ax^2 + bx + 0 = 0x^2 + 0x + 0$$

So $\text{Ker}(\text{Anti}) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

③ $\text{Range}(\text{Anti}) = L(P_2)$ $\qquad$ any $ax+b$ $\qquad L(P_2) = \begin{bmatrix} \frac{1}{2}a \\ b \\ 0 \end{bmatrix} = \underline{\underline{cx^2 + dx + 0}}$

$=$ all poly of $cx^2 + dx + 0$

$$\text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$$

$$\text{span}(x^2, x)$$

(4) $\boxed{\text{Matrix rep. of Anti}}$ in standard. $\text{Anti}\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2}a \\ b \\ 0 \end{bmatrix}$

P_2 from

$$A = \left[\text{Anti}\begin{pmatrix} 1 \\ 0 \end{pmatrix}^{x} \quad \text{Anti}\begin{pmatrix} 0 \\ 1 \end{pmatrix}^{1} \right]$$

$$= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\int (3x+4)\,dx \qquad \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} =$$