Math 511

Q's   Attendance?   Matrices $\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix} = A$ Vectors

Math = toys + rules

$1 \times n \to [1\ 2\ 3\ 4]$ — row vector $= \vec{V}$

$m \times 1 \to \begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$ col. vector $= \vec{V}$

1.3 Matrix Arithmetic

- $A+B$
- $2A$
- AB
- A^T

operators

$$3\begin{bmatrix}-1&0\\1&2\end{bmatrix} = \begin{bmatrix}-3&0\\3&6\end{bmatrix}$$

Must know how to do the operators!

for AB

① Starts: (row vector)(col. vector) = Scalar — 1 real number, 1×1 matrix

$$[1\ 2\ 3\ 4]\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = 1\cdot1 + 2\cdot2 + 3\cdot3 + 4\cdot4 = 1^2+2^2+3^2+4^2$$

② Ax ?

$$\underset{m\times n}{A} = \begin{bmatrix}a_{11} & a_{12} \cdots a_{1n}\\ \vdots & \\ a_{m1} & \cdots a_{mn}\end{bmatrix} = \begin{bmatrix}\vec{a}_{1}\\ \vec{a}_{2}\\ \vdots\\ \vec{a}_{m}\end{bmatrix} = [a_1\ a_2 \cdots a_n]$$

$$Ax = [a_1\ a_2 \cdots a_n]\begin{bmatrix}x_1\\x_2\\ \vdots\\x_n\end{bmatrix} = x_1a_1 + x_2a_2 + \cdots + x_na_n$$

$$A\begin{bmatrix}3\\4\\1/2\end{bmatrix} = 3a_1 + 4a_2 + \tfrac{1}{2}a_3$$

$$Ax = \begin{bmatrix} \vec{a}_1 \\ \vdots \\ \vec{a}_m \end{bmatrix} x = \begin{bmatrix} \vec{a}_1 x \\ \vec{a}_2 x \\ \vdots \\ \vec{a}_m x \end{bmatrix}$$

③ $$\underset{m \times k \;\; k \times n}{\boxed{AB}} = A \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}$$

$$= \begin{bmatrix} Ab_1 & Ab_2 & \cdots & Ab_n \end{bmatrix}$$

$$= \begin{bmatrix} \vec{a}_1 b_1 & \cdots & \vec{a}_1 b_n \\ \vec{a}_2 b_1 & & \vdots \\ \vdots & & \\ \vec{a}_m b_1 & & \vec{a}_m b_n \end{bmatrix} = \begin{bmatrix} \underline{\vec{a}_i b_j} \end{bmatrix}$$

$$\underbrace{\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -1 & 1 \end{bmatrix}}}_{2\times 2} = \underset{2\times 2}{\begin{bmatrix} -2 & 13 \\ 13 & 13 \end{bmatrix}}$$

$\boxed{Q's}$ 6c) 1,2

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 - x_2 - x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right]$$

$\boxed{Nr_2 = r_1 - r_2}$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 0 \\ 0 & \boxed{2} & 2 & 0 \end{array}\right] \xrightarrow{\boxed{Nr_2 = \frac{1}{2} r_2}} \left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \end{array}\right] \xrightarrow{\boxed{Nr_1 = r_1 - r_2}} \left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 0 \\ 0 & \boxed{1} & 1 & 0 \end{array}\right]$$

row ech.

gauss elim

reduced row ech.

gauss - jordan

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \to x_1 = 0 \\ \to x_2 + x_3 = 0 \end{array}$$

x_3 is free, so $\boxed{x_3 = \alpha}$

$x_2 + \alpha = 0$

$x_2 = -\alpha$

$$\boxed{\begin{array}{l} x_1 = 0 \\ x_2 = -\alpha \\ x_3 = \alpha \end{array}}$$

$$\begin{bmatrix} 0 \\ -\alpha \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 - x_2 - x_3 = 0 \end{cases}$$

$x_1 = 0$

$x_2 = -\alpha$

$x_3 = \alpha$

α = anything

ex $\alpha = \pi$

$$\begin{cases} 0 + -\pi + \pi \stackrel{?}{=} 0 & \text{true} \\ 0 - (-\pi) - (\pi) \stackrel{?}{=} 0 & \text{true} \end{cases}$$

1.2 #15

int. #1 $\quad x_1 + 380 = x_2 + 430$

or $\boxed{x_1 - x_2 = 50}$

int #2 $\quad 420 + 450 = x_1 + x_4$

or $\boxed{x_1 + x_4 = 870}$

#3 $\quad x_2 + 540 = x_3 + 420$

$\boxed{x_2 - x_3 = -120}$

#4 $\quad x_3 + 470 = 400 + 420$

$\boxed{x_3 = 350}$

$$\begin{array}{r} 420 \\ -70 \end{array}$$

$$\rightarrow \left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 50 \\ 1 & 0 & 0 & 1 & 870 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 50 \\ 0 & \boxed{1} & 0 & 1 & 820 \\ 0 & 1 & -1 & 0 & -120 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right]$$

$Nr_3 = r_2 - r_3$

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 50 \\ 0 & \boxed{1} & 0 & 1 & 820 \\ 0 & 0 & 1 & 1 & 940 \\ 0 & 0 & 1 & 0 & 350 \end{array}\right] \sim \left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 50 \\ 0 & \boxed{1} & 0 & 1 & 820 \\ 0 & 0 & \boxed{1} & 0 & 350 \\ 0 & 0 & 1 & 1 & 940 \end{array}\right]$$

$Nr_4 = r_4 - r_3$

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & 0 & 0 & 50 \\ 0 & \boxed{1} & 0 & 1 & 820 \\ 0 & 0 & \boxed{1} & 0 & 350 \\ 0 & 0 & 0 & \boxed{1} & 590 \end{array}\right] \sim \left[\begin{array}{cccc|c} \boxed{1} & 0 & 0 & 0 & 280 \\ 0 & \boxed{1} & 0 & 0 & 230 \\ 0 & 0 & \boxed{1} & 0 & 350 \\ 0 & 0 & 0 & \boxed{1} & 590 \end{array}\right]$$

Jordan

row ech.

Gauss elim