Math 511

row op = Matrix

systems

Matrix Algebra

System $\xrightarrow{\text{Solve}}$ ans

$x^2 = 4$

$x^2 - 4 = 4 - 4$

1.5

System → aug. matrix → algebra $\boxed{Ax = b}$

Solve

row ops — swap

ans

E type 1 $= [\quad]$

E type 2 $= [\quad]$

E type 3 $= [\quad]$

$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$

$x = [\quad]$

$[\, I \mid \text{ans} \,]$

$\boxed{E_2 E_1} A x = E_2 E_1 b$

Find A^{-1}

#1

$$[A \mid I]$$

row op

$$[I \mid A^{-1}]$$

$$AX = I$$

$$(E_k \cdots E_2 E_1) A X = \underbrace{E_k E_{k-1} \cdots E_1 I}_{A^{-1}}$$

#2

$$[A \mid B]$$

row ops

#2

$$[I \mid A^{-1}B]$$

#3

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \qquad A^{-1} \stackrel{?}{=} \begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) $= I$

b) $\boxed{Ax = b}$

$A^{-1}Ax = A^{-1}b$

$Ix = A^{-1}b$

$\boxed{x = A^{-1}b}$

(i) $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

R

(ii) $b =$

(iii)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 3 & 3 & 4 & 0 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$E_2 E_1$ $\quad$ E_1

$$\begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_2 + (-3) r_1 = N r_2$

type 3

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ +3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \\ 0 & 3 & 1 & \\ 0 & 2 & 1 & \end{array}\right]$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ +2 & 0 & 1 \end{bmatrix}$$

$r_3 + (-\frac{2}{3}) r_2 = N r_3$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & +2/3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \\ 0 & 3 & 1 & \\ 0 & 0 & 1/3 & \end{array}\right]$$

Gauss

U

$$E_3 E_2 E_1 A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$x^3 = y$

$x = -\frac{1}{2}z$

$$\boxed{A = L \cdot U}$$

1.5 Know ① $\boxed{\text{Find } A^{-1}}$ $[A \mid I] \rightsquigarrow [I \mid A^{-1}]$

② Use same tech $[A \mid U] \rightsquigarrow [I \mid A^{-1}U]$

#12

③ Know all E operators E^{-1} operators

$A = LU$

10 f

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 5 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

swap

$$E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 5 & 1 & 0 & 0 \end{array}\right] \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right] \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -5 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right] \quad E_4 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -5 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -1 & 0 & 2 \end{array}\right] \quad E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

check: $AA^{-1} \stackrel{?}{=} I$ $\qquad$ $A^{-1}A \stackrel{?}{=} I$

show $E_5 E_4 E_3 E_2 E_1 A = A^{-1}$

1.6 Partitioned Matrix

"Same"

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & 5 \end{bmatrix}$$

$$\left[\begin{array}{c|c} A(1\times) & B \\ \hline C & D \end{array}\right] + \left[\begin{array}{c|c} 8(1\times) & F \\ \hline G & H \end{array}\right] = \begin{bmatrix} A(1\times)+8(1\times) & B+F \\ C+G & D+H \end{bmatrix}$$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \cdot \left[\begin{array}{c|c} E & F \\ \hline G & H \end{array}\right] = \begin{bmatrix} AE+BG & AF+BH \\ & \text{etc} \end{bmatrix}$$

1.6 1c

$$[A \mid I]^T [A \mid I]$$

$$\begin{array}{c c} & \begin{array}{cc} \text{col 1} & \text{col 2} \end{array} \\ \begin{array}{c} \text{row 1} \to \\ \text{row 2} \to \end{array} & \begin{bmatrix} A \\ \hline I \end{bmatrix} \begin{bmatrix} A & I \end{bmatrix} \end{array} = \begin{bmatrix} AA & AI \\ IA & I \cdot I \end{bmatrix}$$

$2 \times 1 \qquad 1 \times 2 \qquad 2 \times 2$

$$= \left[\begin{array}{c|c} A^2 & A \\ \hline A & I \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

7 5

ex $\quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 1 & 2 \\ 15 & 22 & 3 & 4 \\ 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix}$$

$4 \times 2 \qquad 2 \times 4 \qquad 4 \times 4$

21/22 Find det(A)

given A

does A^{-1} exist