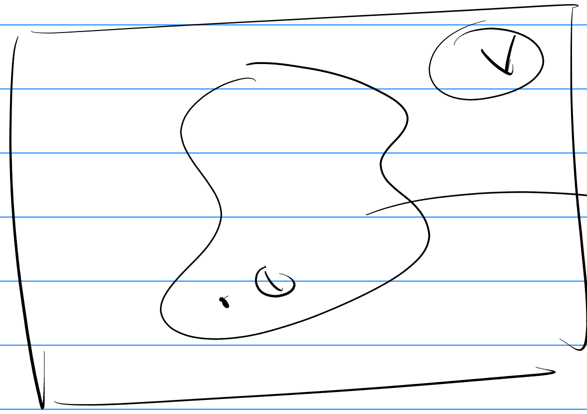
Math 511

① "zero" object (Additive Identity)

Subspaces:



S

$\mathbb{R}^3$ $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

P_3 $0 + 0x + 0x^2 = 0$

Def: S is a subspace if $S \subseteq V$ ← vector space

and S is itself a vector space

to show

① show $0 \in S$

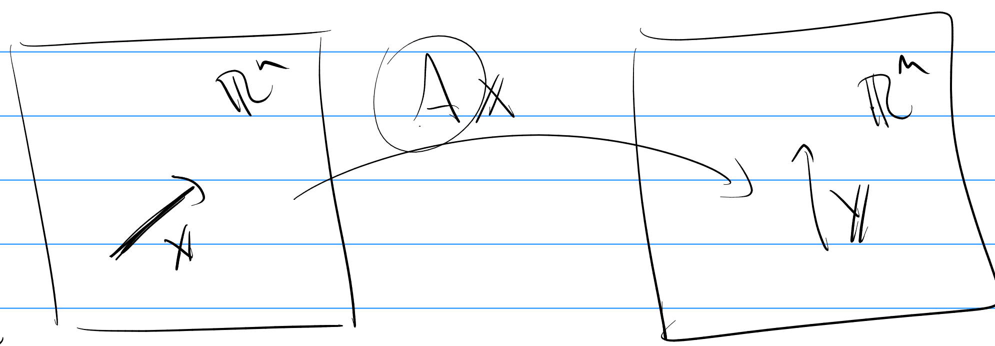
② $\alpha v \in S$

③ $v_1 + v_2 \in S$

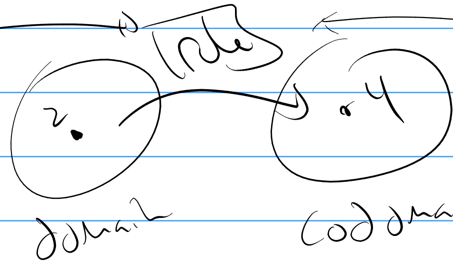
Null Space

$\boxed{A x = y}$

$m \times n$ $n \times 1$ $m \times 1$



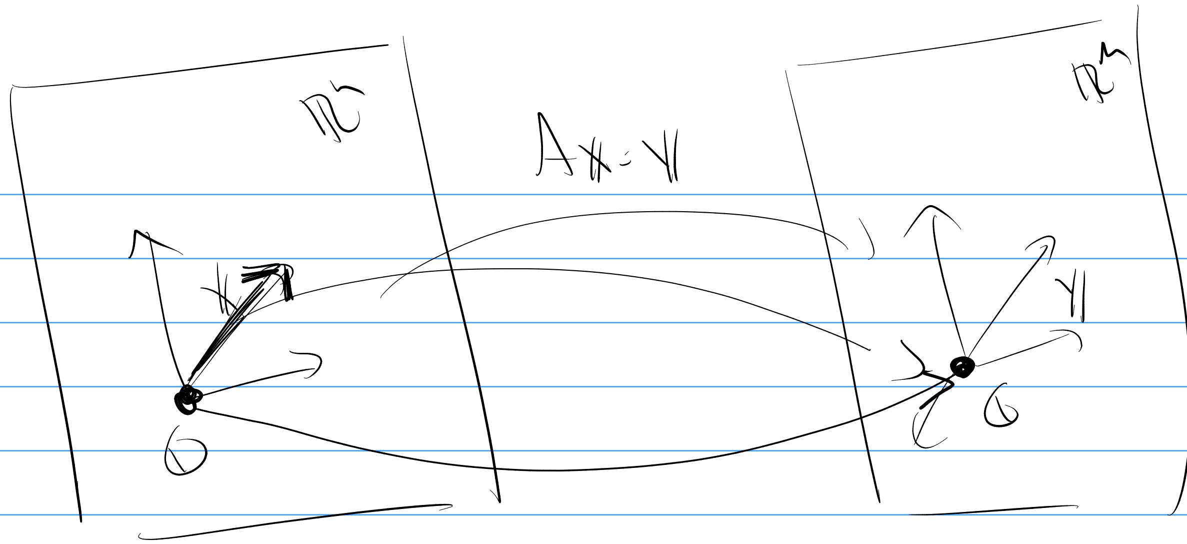
Function



$f(x) = x^2$

$f(2) = 4$

$2 \xrightarrow{f} 4$



domain codomain

Note: $A\mathbb{x} = 0$

what $\mathbb{x}$'s in $\mathbb{R}^n$ go to 0 in $\mathbb{R}^m$

$N(A) = \{ \mathbb{x} \mid \text{all } \mathbb{x} \in \mathbb{R}^n \text{ such that } A\mathbb{x} = 0 \}$

all sol'n's to homogeneous system

① is $0 \in N(A)$?

yes b/c $A\,0_{\mathbb{R}^n} = 0_{\mathbb{R}^m}$

② if $\boxed{\mathbb{v} \in N(A)}$ is $\boxed{\alpha \mathbb{v} \in N(A)}$?

$A\mathbb{v} = 0$

$A(\alpha\mathbb{v}) = \alpha A\mathbb{v}$
$= \alpha 0 = 0$

true

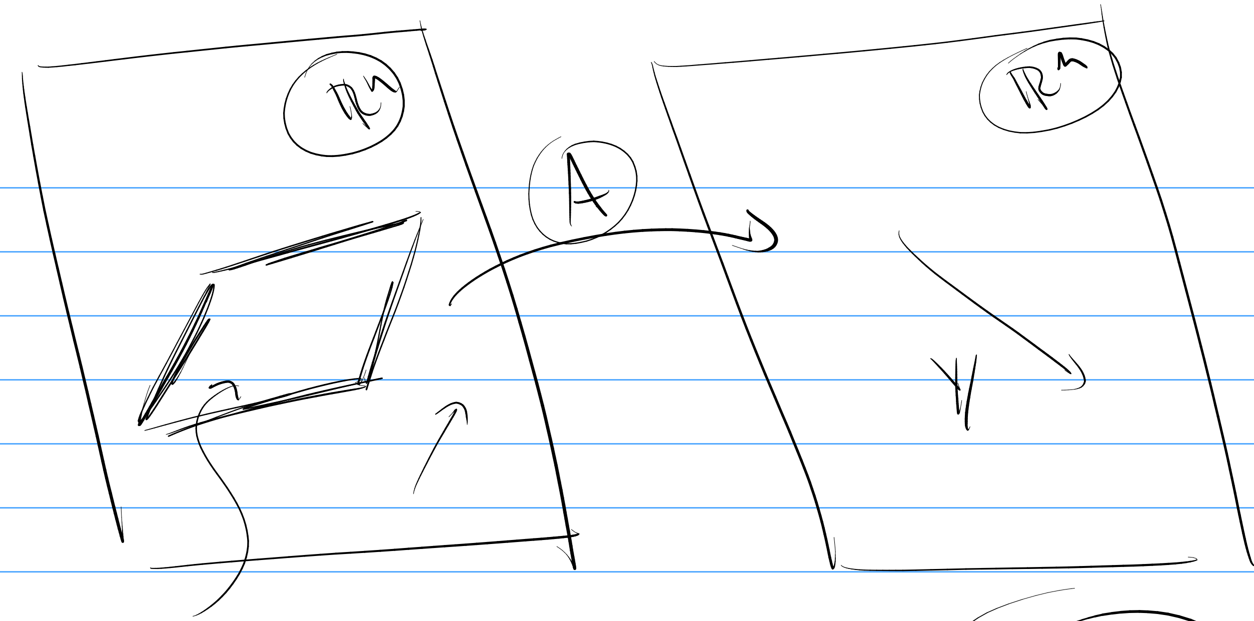
③ if $\mathbb{v}_1, \mathbb{v}_2 \in N(A)$ is $\mathbb{v}_1 + \mathbb{v}_2 \in N(A)$

$A\mathbb{v}_1 = 0 \quad A\mathbb{v}_2 = 0$

so $A[\mathbb{v}_1 + \mathbb{v}_2] = A\mathbb{v}_1 + A\mathbb{v}_2$
$= 0 + 0 = 0$

true

So



$N(A) = \{ x \mid \text{all sol's } x \text{, to } Ax = 0 \}$

3.2 3g

$V = \mathbb{R}^{2\times 2}$ is my vector space

is $S = \{$ set of all A such that A is singular $\}$

- $\det(A) = 0$
- no inv
- non-invertable
- $Ax = 0$ has infinite sols

ex $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\det(A) = 1\cdot 1 - 1\cdot 1 = 0$

so $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \in S$

Same as

$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \ ad - bc = 0 \right\}$

Check for subspace?

① is $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$

$\det(0) = 00 - 00 = 0$

(2) is $\alpha M \in S$ $\quad M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

know $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in S$ $\quad$ show $\alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in S$

$ad - bc = 0$ $\quad \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix}$

check: $\det$ $\quad \det\begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix} = \alpha^2 ad - \alpha^2 bc$

$= \alpha^2 (ad - bc)$

true $\quad = \alpha^2 \cdot 0 = 0$

(3) M_1 and M_2 are singular matrices

is $M_1 + M_2$ singular?

$M_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ $\quad M_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

$\boxed{a_1 d_1 - b_1 c_1 = 0}$ $\quad \boxed{a_2 d_2 - b_2 c_2 = 0}$

what about $M_1 + M_2 = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$

$\det(M_1 + M_2) = (a_1 + a_2)(d_1 + d_2) - (b_1 + b_2)(c_1 + c_2)$

$= \boxed{\cancel{a_1 d_1}} + a_1 d_2 + a_2 d_1 \boxed{+ \cancel{a_2 d_2}} \boxed{- \cancel{b_1 c_1}} - b_1 c_2 - b_2 c_1 \boxed{- \cancel{b_2 c_2}}$

$$\det(M_1 + M_2) = \boxed{a_1 d_2 + a_2 d_1 - b_1 c_2 - b_2 c_1}$$

$$M_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$$

$$a_1 d_1 - b_1 c_1 = 0 \qquad a_2 d_2 - b_2 c_2 = 0$$

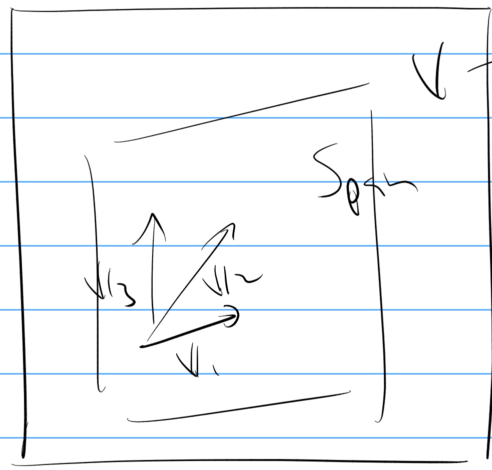
$$M_1 = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \text{ singular} \qquad M_2 = \begin{bmatrix} -3 & 2 \\ 6 & -4 \end{bmatrix} \text{ singular}$$

$$M_1 + M_2 = \begin{bmatrix} -1 & 6 \\ 9 & 2 \end{bmatrix} \qquad \det(M_1 + M_2) = -56 \neq 0$$

not singular

Not a subspace

Linear Independence



a vector space

Subspaces
to know

(1) Null Space
(2) Spans

$\text{Span}(v_1, v_2, v_3)$

$n = \alpha_1 \boxed{\$5} + \alpha_2 \boxed{\$3}$ is all $v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$

span = $(\$0, \$3, \$5, \$6, \boxed{\$8, \$9, \$10}, \$11, \$12, \$13, \$14, \ldots)$

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linear dependent is like $\$8 = 1 \cdot \$5 + 1 \cdot \$3$

why have an \$8 bill

linear ind. is not this

Know

check: $\boxed{\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_k v_k = 0}$

only trivial sol → ind.
non-trivial sol → dep