Math 511

3.2 #6

$$\left[\begin{array}{cccc|c} \overset{x_1}{1} & \overset{x_2}{2} & \overset{x_3}{-3} & \overset{x_4}{-1} & 0 \\ -2 & -4 & 6 & 3 & 0 \end{array}\right] \quad 2r_1 + r_2 = r_2$$

Null space

$A\mathbf{x} = 0$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 2 & -3 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \rightarrow x_1 + 2\alpha - 3\beta = 0 \\ \rightarrow x_4 = 0 \end{array}$$

free free

$x_1 = -2\alpha + 3\beta$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \begin{array}{l} x_2 = \alpha \\ x_3 = \beta \end{array}$$

Null space

$$\mathbf{x} = \begin{bmatrix} -2\alpha + 3\beta \\ \alpha \\ \beta \\ 0 \end{bmatrix} = \alpha \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

as a Span

$$\boxed{N(A) = \operatorname{Span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)}$$

3.4 8d $\quad x_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad x_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} \quad x_3 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$

dep? $\quad X = \begin{bmatrix} 2 & 3 & 2 \\ 1 & -1 & 6 \\ 3 & 4 & 4 \end{bmatrix}$

ind/dep check $\quad \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = 0$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 2 & 0 \\ 1 & -1 & 6 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \xrightarrow{\text{row ops}} \left[\begin{array}{c|c} \quad & \quad \end{array}\right]$$

ind. ind.

soln

$\frac{1}{5} r_2 = r_2 \quad r_3 - r_2 = r_3$

$\frac{1}{7} r_3 = r_3$

$$X = \begin{bmatrix} 2 & 3 & 2 \\ 1 & -1 & 6 \\ 3 & 4 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 6 \\ 2 & 3 & 2 \\ 3 & 4 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 6 \\ 0 & 5 & -10 \\ 0 & 7 & -14 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

lead lead free

reduced row-ech.

dep. eqn's

$4x_1 - 2x_2 = x_3$

$$4\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

$x_1 \qquad x_2 \qquad x_3$

$\text{Rank}(X) = 2$

$\text{nullity}(X) = \dim(N(X)) = 1$

d) know x_1, x_2 are ind. $\quad x_3$ is dep

$\text{Span}(x_1, x_2, x_3) = \text{Span}(x_1, x_2)$ is a plane

(14) Use another way to consider P_n

"long way" $\quad$ Span$(x, x-2)$ in P_3 $\quad (p = a + bx + cx^2)$

if independent → both are needed $\quad$ dim(Span) = 2

$$\alpha_1 x + \alpha_2(x-2) = 0 + 0x + 0x^2$$

$$-2\alpha_2 + (\alpha_1+\alpha_2)x + 0x^2 = 0 + 0x + 0x^2$$

$\alpha_2 = 0$

$\alpha_1 + \alpha_2 = 0 \Rightarrow \alpha_1 = 0$ $\quad$ only trivial sol.

→ ind.

→ dim(Span) = $\boxed{2}$

$$1{,}034 = 1\cdot\boxed{1000} + 0\cdot\boxed{100} + 3\cdot\boxed{10} + 4\cdot\boxed{1}$$

$$\cancel{4\cdot 1 + 2\cdot 100 + 1\cdot 1000 + 3}\cdot 10$$

$\mathbb{R}^n$

$P_2 = x - 2$ $\quad$ (n) $\quad\quad$ $p = a + bx + cx^2$

$P_2 = -2 + 1\cdot x + 0\cdot x^2$ $\quad\quad$ $p = cx^2 + bx + a$

$P_2 = 0\cdot x^2 + 1\cdot x - 2$

$$P_2 = \begin{array}{c l} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} & \begin{array}{l} \text{const} \\ x \\ x^2 \end{array} \end{array} \qquad p = \begin{array}{c l} \begin{bmatrix} a \\ b \\ c \end{bmatrix} & \begin{array}{l} \text{const} \\ x \\ x^2 \end{array} \end{array} \text{ for } a + bx + cx^2$$

So we could do the following

14a) P_3 $\quad 2x,\ x-2$

(15) rather $\mathbb{R}^3$ $\quad \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

use all the normal matrix knowledge

Dimension vs number of vectors

Span(v_i) Subspace built by v_i

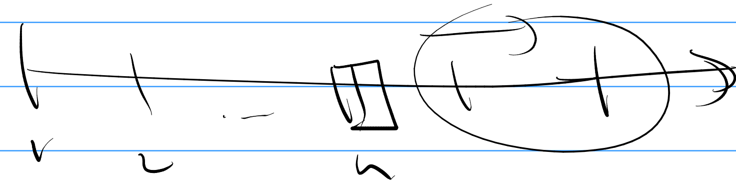
& $V = \text{Span}(v_i)$

call them spanning

exact number of ind. vectors that span V.

① $\dim(V) = n$ so $\boxed{\text{any}}$ (n) ind. vectors are a basis

② what about $m > n$ (more vectors)



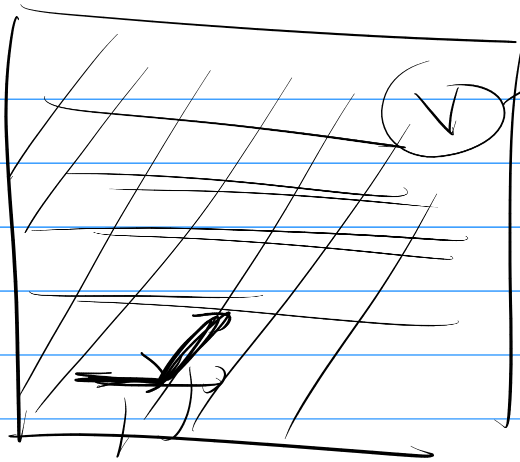
how many vectors

ex) $\mathbb{R}^3$ $\quad \dim(\mathbb{R}^3) = 3$

given v_1, v_2, v_3, v_4, v_5

$$\dim\left(\text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)\right)$$

$= 1$



Vector Space

? objects

? ops $\alpha_1 v_1 + \alpha_2 v_2$

tear them down

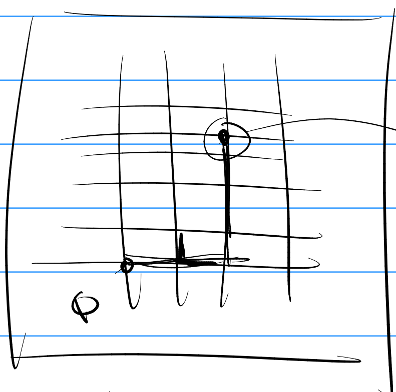
&

? dimension

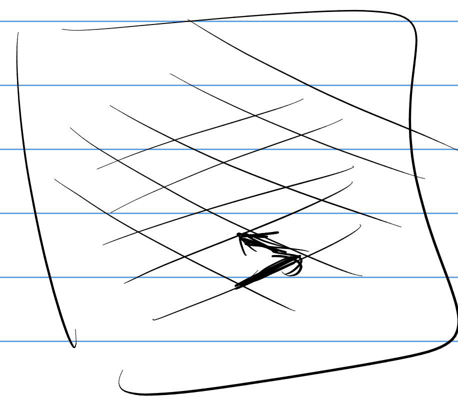
only # of vectors to go everywhere

? Basis

it's ok to have more than one basis



$c_1 b_1 + c_2 b_2 \vdots + c_k b_k$



Basis $b_1, b_2, \dots, b_k$

Basis $d_1, d_2, \dots, d_k$

$$v = [b_1\ b_2 \cdots b_k] \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}$$

$$\text{So } v = [v]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}_B$$

$$v = [v]_D = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_k \end{bmatrix}_D$$

coordinates

Change of basis

① $[x]_E = B[x]_B$

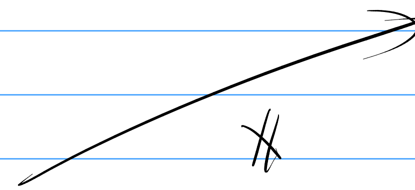
E: standard

$[x]_B$: vector x using basis $B = \{b_1, b_2 \dots b_k\}$ coord.

② $[x]_B = B^{-1}[x]_E$

Basis $B = \{b_1 \dots b_k\}$

$D = \{d_1 \dots d_k\}$



$$[x]_B = \boxed{B^{-1} D}\underbrace{[x]_D}_{[x]_E}$$

$[x]_D$: x in coord. of D

$$\boxed{S[x]_S = [x]_E}$$