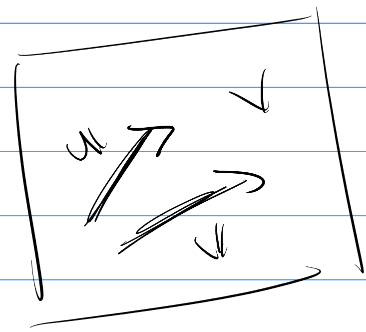
Math 511

Q's



orthonormal basis $U = [u_1, u_2, u_3]$

$u = 1\cdot u_1 + 2u_2 + 2u_3 = \begin{bmatrix}1\\2\\2\end{bmatrix}_U$

$v = 1\cdot u_1 + 0\cdot u_2 + 7u_3 = \begin{bmatrix}1\\0\\7\end{bmatrix}_U$

$\langle u, v\rangle = [1\ 2\ 2]\begin{bmatrix}1\\0\\7\end{bmatrix} = \boxed{15}$

$\|u\| = \sqrt{\langle u, u\rangle} = \sqrt{[1\ 2\ 2]\begin{bmatrix}1\\2\\2\end{bmatrix}} = \underline{\sqrt{9}} = 3$

$\|v\| = \sqrt{[1\ 0\ 7]\begin{bmatrix}1\\0\\7\end{bmatrix}} = \underline{\sqrt{50}}$

etc...

5.5 4a $\quad x_1 = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \quad x_2 = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \quad \mathbb{R}^2$

<u>orthonormal?</u> $\quad \|x_1\| = \sqrt{[\cos\theta\ \sin\theta]\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}} = \sqrt{1} = 1$

(1) <u>check length</u>

$\|x_2\| = \dots$ do it

(2) check $\perp$ $\quad [\cos\theta\ \sin\theta]\begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} = 0$ yes

4b $\quad y$ in $\mathbb{R}^2 \quad y = \begin{bmatrix}c_1\\ c_2\end{bmatrix}_X \quad X = [x_1\ x_2]$

Note:

4c

$c_1^2 + c_2^2 = \|y\|^2$
$= y_1^2 + y_2^2$

ok.. $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

4b

$\underline{\underline{c_1}} = \langle y, x_1 \rangle = y^T x_1 = [y_1 \; y_2] \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

$= y_1 \cos\theta + y_2 \sin\theta$

$\underline{\underline{c_2}} = \langle y, x_2 \rangle = y^T x_2 = [y_1 \; y_2] \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$

$= -y_1 \sin\theta + y_2 \cos\theta$

4c show $\|y\|^2 = c_1^2 + c_2^2 = y_1^2 + y_2^2$

b/c $\|y\|^2 = \langle y, y \rangle =$ use coord & orthonormal basis

$= [c_1 \; c_2] \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = c_1^2 + c_2^2$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ orthonormal

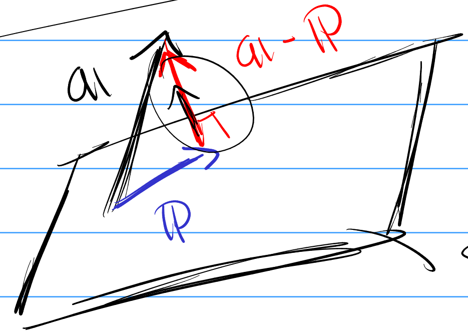
standard basis coords

$= [y_1 \; y_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = y_1^2 + y_2^2$

Heart of Gram Schmidt

V

1



Subspace

$S = \mathrm{Span}(q_1, q_2, \ldots, q_k)$

orthonormal

$P \in S$

$a_1 - P \perp \{q_1, q_2, \ldots, q_k\}$

2 $P = \begin{bmatrix} \langle a_1, q_1 \rangle \\ \langle a_1, q_2 \rangle \\ \langle a_1, q_k \rangle \end{bmatrix} Q$

(3) $\dfrac{a - p}{\|a - p\|}$ ← a) orthogonal to all $q_1, q_2, \ldots$

b) length is 1

call $\dfrac{a - p}{\|a - p\|} = q_{k+1}$

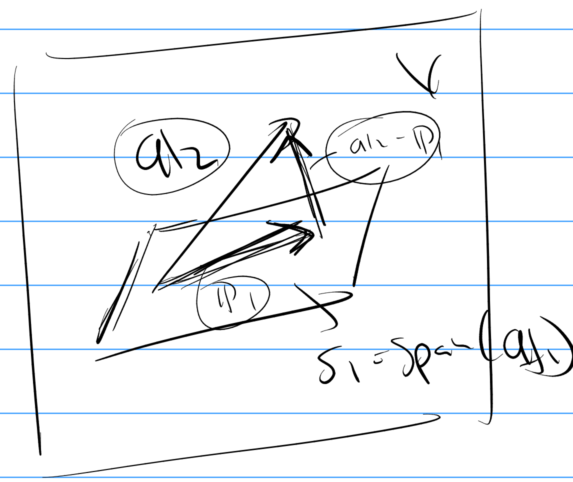
then $q_1, q_2, \ldots, q_k, q_{k+1}$

is also orthonormal

$a_1, a_2, \ldots, a_n$ not orthonormal

Basis Step

step 1 $q_1 = \dfrac{a_1}{\|a_1\|}$

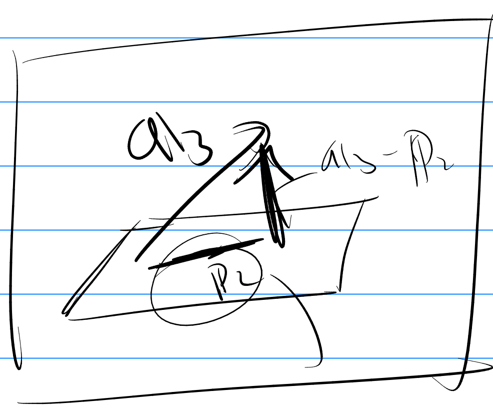


Inductive

step 2 $p_1 = \left[\langle a_2, q_1 \rangle \right]$

$\dfrac{a_2 - p_1}{\|a_2 - p_1\|} = q_2$

step 3 $p_2 = \begin{bmatrix} \langle a_3, q_1 \rangle \\ \langle a_3, q_2 \rangle \end{bmatrix}$



(ch) $\dfrac{a_3 - p_2}{\|a_3 - p_2\|} = q_3$

$S_2 = \text{Span}(q_1, q_2)$

Note: $A = [a_1 \; a_2 \cdots a_n]$

$Q = [q_1 \; q_2 \cdots q_n]$

$$[a_1 \; a_2 \cdots a_n] = [q_1 \; q_2 \cdots q_n] \begin{bmatrix} \|a_1\| & \langle a_2, q_1 \rangle & \langle a_3, q_1 \rangle & \cdots \\ 0 & \|a_2 - p_1\| & \langle a_3, q_2 \rangle & \\ 0 & 0 & \|a_3 - p_2\| & \\ \vdots & \vdots & 0 & \\ 0 & 0 & \vdots & \end{bmatrix}$$

orthogonal matrix

$A = QR$ ← upper triangular

So (a) solve $Ax = b$

$x = A^{-1} b$

or $Ax = b$

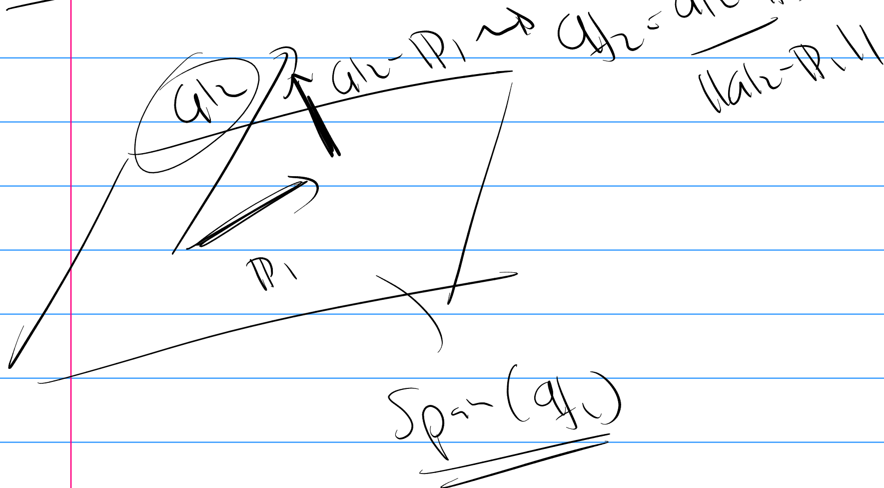
$QRx = b$ $\quad Q^{-1} = Q^T$

orthogonal matrix!

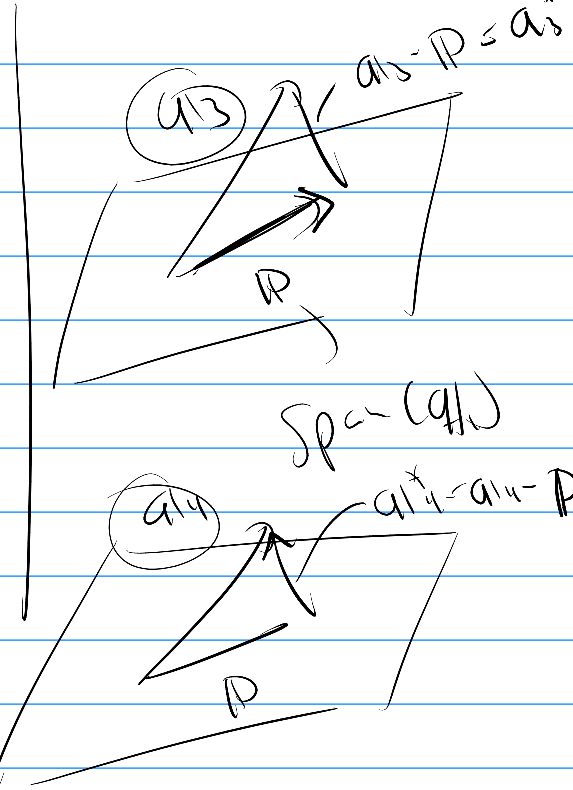
$\boxed{Rx = Q^T b}$

upper triangular → solve by back solve

Modified Gram Schmidt



modified (a_i)



$$\underline{gm}(x_1, x_2, x_3, x_4) = \boxed{(x_1 \cdot x_2 \cdot x_3 \cdot x_4)^{1/4}} \leftarrow$$

$$\boxed{\underline{\text{Same}}} = \exp\left(\ln (x_1 \cdot x_2 \cdot x_3 \cdot x_4)^{1/4}\right)$$

$$= \boxed{\exp\left(\frac{\ln(x_1) + \ln(x_2) + \ln(x_3) + \ln(x_4)}{4}\right)} \leftarrow$$