Math 511

Section 1.6 — Partitioned Matrices

heart of matrix multiply is (row)(column)

before: $A\vec{v}$ ($m \times n$ $n \times 1$) $= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} = a_{11}v_1 + a_{12}v_2 + \cdots + a_{1n}v_n$

($a_{12}v_2$: vector, scalar)

$= v_1 a_{11} + v_2 a_{12} + \cdots + v_n a_{1n}$

($v_2 a_{12}$: typical (correct?) way to write scalar · vector.)

$\begin{bmatrix} a_{ij} \end{bmatrix}$

or $A\vec{v} = \begin{bmatrix} \vec{a}_{11} \\ \vec{a}_{12} \\ \vdots \\ \vec{a}_{1n} \end{bmatrix} \vec{v} = \begin{bmatrix} \vec{a}_{11}v_1 \\ \vec{a}_{12}v_1 \\ \vdots \\ \vec{a}_{1n}\vec{v} \end{bmatrix}$

Notice: these are using vectors for A's insides...

But we do the same as if they are scalars

(similar technique)

(ex) $Av = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = v_1 a_1 + \cdots + v_n a_n$

(ex) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = (1)(1) + (2)(-1) + (3)(0) + (4)(1)$

$=$

Partitioned Matrices

Before: $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} col_1 & col_2 & \cdots & col_n \end{bmatrix}$

$= \begin{bmatrix} row_1 \\ row_2 \\ \vdots \\ row_m \end{bmatrix}$

Now? what about any partitioning?

(ex) $A = \begin{bmatrix} -1 & 0 & 1 & 2 & 3 \\ 1 & 2 & 1 & 3 & 4 \\ -1 & 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 2 & 3 \\ \hline 1 & 2 & 1 & 3 & 4 \\ \hline -1 & 0 & 0 & 1 & 2 \end{bmatrix}$

$= \left[\begin{array}{c|c|c|c|c} -1 & 0 & 1 & 2 & 3 \\ \hline 1 & 2 & 1 & 3 & 4 \\ \hline -1 & 0 & 0 & 1 & 2 \end{array}\right] = \left[\begin{array}{cccc|c} -1 & 0 & 1 & 2 & 3 \\ \hline 1 & 2 & 1 & 3 & 4 \\ \hline -1 & 0 & 0 & 1 & 2 \end{array}\right]$

1×2

$= \left[\begin{array}{cc|c|cc} -1 & 0 & 1 & 2 & 3 \\ \hline 1 & 2 & 1 & 3 & 4 \\ \hline -1 & 0 & 0 & 1 & 2 \end{array}\right] = \left[\begin{array}{cc|c|c} A_{11} & A_{12} & A_{13} \\ \hline A_{21} & A_{22} & A_{23} \end{array}\right]$

2×2

Currently sized blocks

Now: Operations are all the same as long as the sizes you picked match up.

ex

2x2 partition

$\begin{bmatrix} 1 & 2 & | & 1 & -1 & 0 \\ 3 & 4 & | & 1 & 1 & 1 \\ \hline 1 & -1 & | & 2 & 4 & 5 \end{bmatrix}$ + $\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ \hline 1\times 2 & & | & 1\times 3 \end{bmatrix}$ = $\begin{bmatrix} 2 & 1 \\ 3 & 5 \\ \hline & & | & \end{bmatrix}$

2x3 — 2x2 partition

3x5 $\begin{bmatrix} 1 & 2 & | & 1 & -1 & 0 \\ 3 & 4 & | & 1 & 1 & 1 \\ \hline 1 & -1 & | & 2 & 4 & 5 \end{bmatrix}$ · 5x10 $\begin{bmatrix} 0 & | & 2 & | \\ 0 & | & & | \end{bmatrix}$ 2x3

2x2 partition — 2x (any size) | partition

2x2 $\begin{bmatrix} A_{11} & | & A_{12} \\ \hline A_{21} & | & A_{22} \end{bmatrix}$ 2x3 — 1x2, 1x3

but

2xK $\begin{bmatrix} B_{11} & B_{12} & | & B_{13} \\ B_{21} & B_{22} & | & B_{23} \end{bmatrix}$ 3xK — 2x3

Same col's #

2x2 — 2xK, 2x3, 3xK

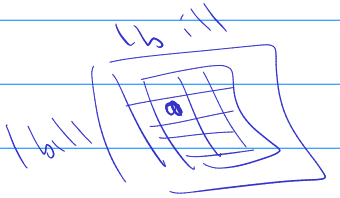
ex = $\begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & | \\ & | \end{bmatrix}$

2x3

result

Why? Very Very Very Large Matrices

(vs) limited Memory.

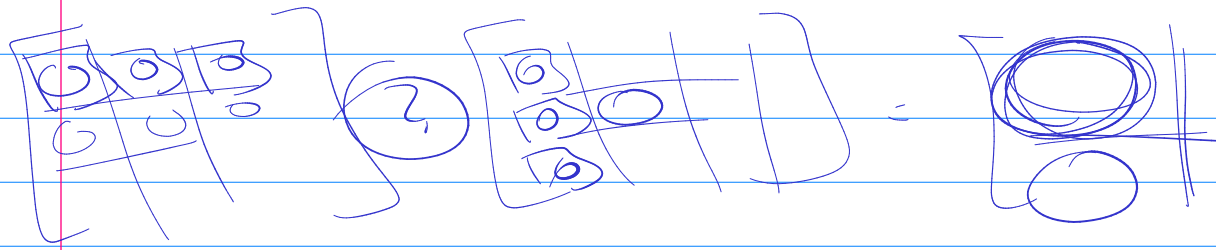


(ex) $10^9 \times 10^9$ Matrix = 10^{18} elements (1 billion)

64 bit system

8 bytes / element

so 8×10^{18} bytes



(or) $\sim 10^{19}$ bytes

or $\sim$ 10,000,000,000,000,000,000 bytes of data

or $\sim$ 10,000,000 terabytes to hold data.

for 1 million $\times$ 1 million

$10^6 \times 10^6 = 10^{12} \sim 10^{13}$

10,000,000,000,000 $\rightarrow$ 4 terabytes to hold data