# Math 242

Q's/ Webassign - extensions? (You may need to remind me)

1.5 #36 $\lim_{x\to\pi^-} \cot x$ vs $\lim_{x\to\pi^-} \csc x$

$\boxed{\sin x,\ \cos x}$ Identities

hyp, opp, adj, $x$

$\boxed{adj^2 + opp^2 = hyp^2}$

$\boxed{\sin^2 x + \cos^2 x = 1}$

$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

$\sec x = \dfrac{1}{\cos x}$ $\csc x = \dfrac{1}{\sin x}$

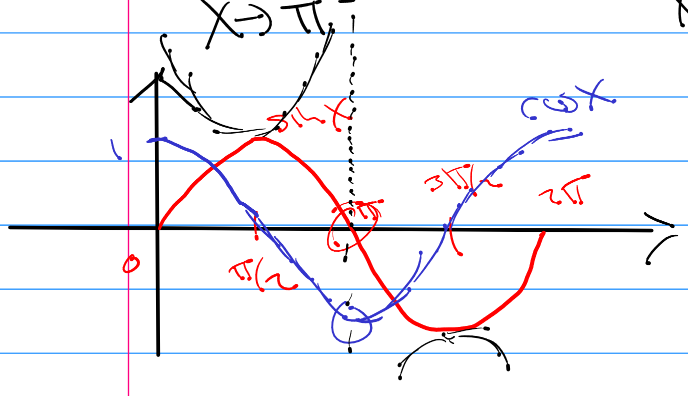
$\to \lim_{x\to\pi^-} \cot x = \lim_{x\to\pi^-} \dfrac{\cos x}{\sin x}$ vs $\lim_{x\to\pi^-} \csc x = \lim_{x\to\pi^-} \dfrac{1}{\sin x}$

Concept $\boxed{\dfrac{\text{top} \to \text{const.}}{\text{bottom} \to 0} \longrightarrow \pm\infty}$

$\lim_{x\to\pi^-} \dfrac{\cos x}{\sin x} = \lim_{x\to\pi} \dfrac{-1}{\sin x \to 0} = -\infty$

but positive

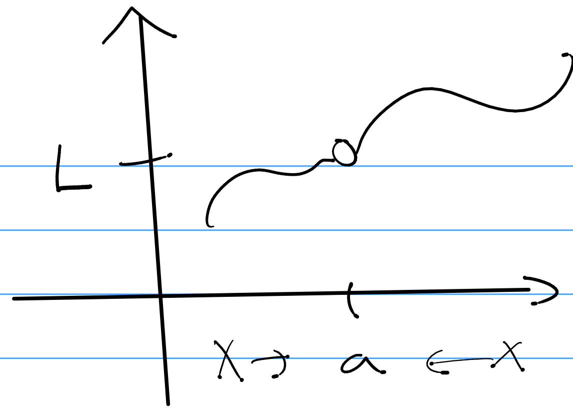
$\lim_{x\to\pi^+} \dfrac{1}{\sin x \to 0}$ but neg. $= -\infty$




vs $\lim_{x\to\pi^-} \dfrac{1}{\sin x \to 0} = +\infty$

but pos.

# Limits




Intuitive version

$\lim_{x \to a^-} f(x) = L$ means ...

$\lim_{x \to a^+} f(x) = L$ means ...

$\lim_{x \to a} f(x) = L$ means ..

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"Know" $\lim_{x \to a} c = c$, $\lim_{x \to a} x = a$

Still Intuitive (+) limit laws. $+, -, *, \div, (\ )^p$

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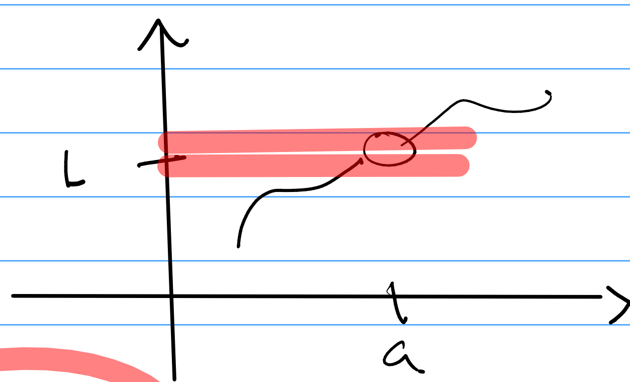
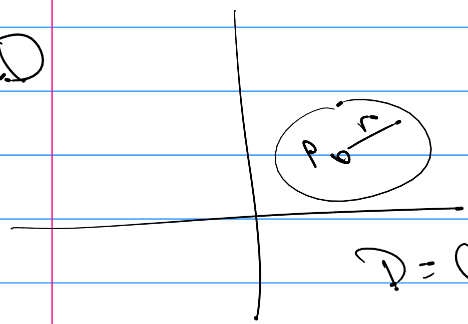
Better? $\lim_{x \to a} f(x) = L$




means

as x gets close to a then f(x) gets close to L "gets close" into <u>math</u> (use a ball)

2D

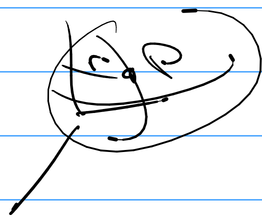



$P = (x_0, y_0)$

close to P is inside a circle of radius r

$$\left((x-x_0)^2 + (y-y_0)^2\right)^{1/2} < r$$

3D

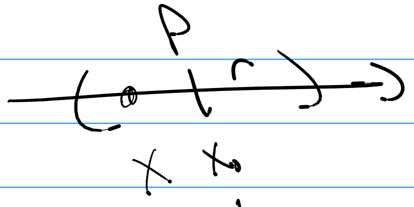



$P = (x_0, y_0, z_0)$

close to P is inside the ball

$$\left((x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right)^{1/2} < r$$

1D




$|x-x_0| = \text{dist}$

close to P is inside the "ball" (1D ball is an interval)

$$\left((x-x_0)^2\right)^{1/2} < r$$

$$|x-x_0| < r$$

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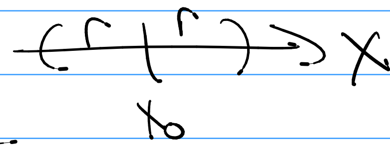
"gets close" becomes distance < given value

ex




$|x-x_0| < r$

as an interval x is in ...

$$\boxed{|x-x_0| < r} \to -r < x - x_0 < r$$

$$\to \boxed{x_0 - r < x < x_0 + r}$$

so x is in $(x_0 - r, x_0 + r)$ Interval notation back to $\lim\limits_{x \to a} f(x) = L$

x gets close to a

**1st** gets close with $|x - a| < r$ type notation

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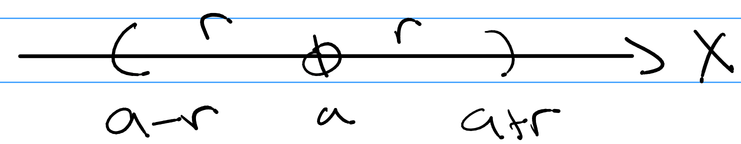
**2nd** $x \neq a$ for a limit.

x gets close to a, but $x \neq a$

$$0 < |x-a| < r$$ it means..

**Interval**




$(a-r, a) \cup (a, a+r)$

left side   right side

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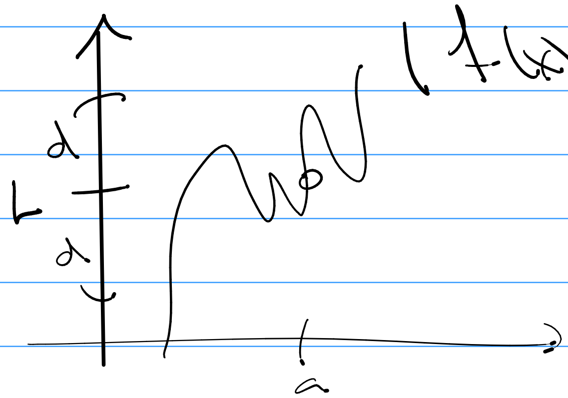
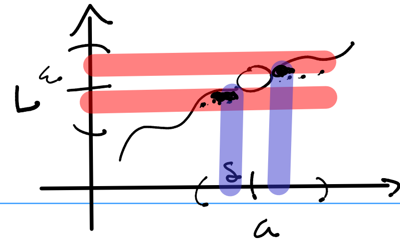
**3rd** f(x) gets close to L

$$|f(x) - L| < d$$



all together: $\lim_{x \to a} f(x) = L$




as x gets close to a, then f(x) gets close to L

**Def** $\lim_{x \to a} f(x) = L$

"given to limit you" "for you to find given ε"

for any $\varepsilon > 0$ there is a $\delta > 0$ such that

if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$

x's are within δ distance of a — then — f(x) is within ε distance of L

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Notation: **Def** $\lim_{x \to a} f(x) = L$ is

$$\boxed{\forall \varepsilon > 0},\ \exists \delta > 0 \ni,\ (0 < |x-a| < \delta) \rightarrow (|f(x) - L| < \varepsilon)$$

for all — there exists — such that — if — , then

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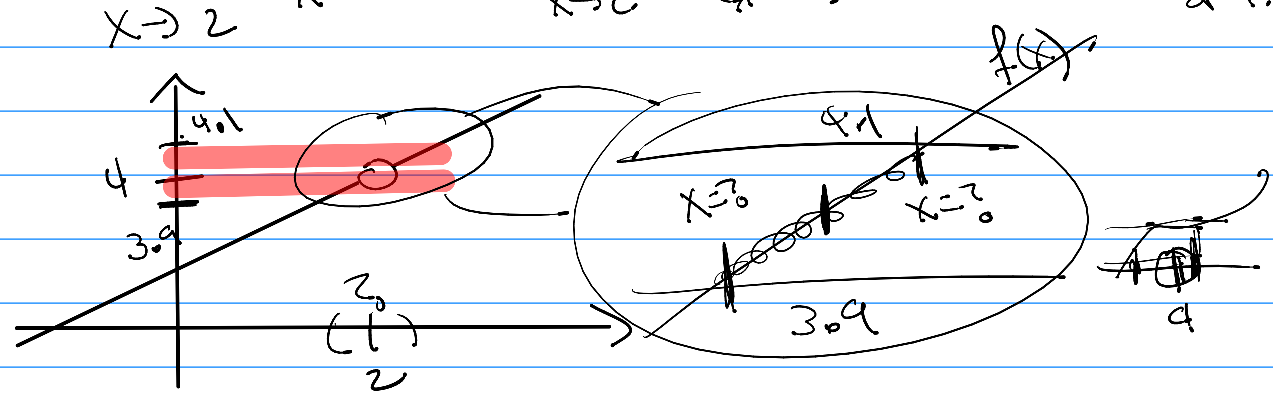
given $\varepsilon = 0.1$, find $\delta = ?$

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4$$

Intuitive version of 1.6

ε, δ

ε = 0.1



$\lim_{x \to 2} x+2 = 4$ given $\varepsilon > 0$ pick a $\delta > 0$

So that $\boxed{0 < |x-2| < \delta} \to |(x+2)-4| < \varepsilon$

Note: Direct proof is given the left side

show the right side must follow.

goal: Start at left: $0 < |x-2| < \delta$

{do algebra correctly} ← Valid: replace $\delta$, (correctly) add, sub, mult, divide
use inequalities

until right side is true $|(x+2)-4| < \varepsilon$

(1) Scratch work 1st to study the right side

sc $|(x+2)-4| < \varepsilon$

→ Same as $|x-2| < \varepsilon$ (compare to left $0 < |x-2| < \delta$)

obvious let $\delta = \varepsilon$

(2) Proof:

$0 < |x-2| < \delta$ let $\delta = \varepsilon$

so $0 < |x-2| < \varepsilon$, So $0 < |x \underbrace{+2-4}_{=(-2)}| < \varepsilon$

$\therefore$ $|(x+2)-4| < \varepsilon$ is true

$|f(x) - L| < \varepsilon$

Ex $\displaystyle\lim_{x \to 1} \frac{2+4x}{3} = \frac{2+4(1)}{3} = 2$

$\boxed{\text{prove}}$ means find $\delta$ so that

$$0 < |x-1| < \delta \rightarrow \left|\left(\frac{2+4x}{3}\right) - 2\right| < \varepsilon$$

(1) mess with right side $\boxed{\left|\left(\frac{2+4x}{3}\right) - 2\right| < \varepsilon}$

i.e. $|(2+4x) - 6| < 3\varepsilon$, i.e. $|4x-4| < 3\varepsilon$

i.e. $|x-1| < \frac{3}{4}\varepsilon$

with left side $0 < |x-1| < \delta$ pick $\delta = \frac{3}{4}\varepsilon$!

(2) $\boxed{\text{Proof}}$ $0 < |x-1| < \delta$ let $\delta = \frac{3}{4}\varepsilon$

so $0 < |x-1| < \frac{3}{4}\varepsilon$

then $4|x-1| < 3\varepsilon$, i.e. $|4x-4| < 3\varepsilon$

i.e. $|(2+4x) - 6| < 3\varepsilon$

i.e. $\left|\frac{2+4x}{3} - 2\right| < \varepsilon$

$\therefore |f(x) - L| < \varepsilon$ $\boxed{\text{done}}$

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$$\lim_{x \to 2} (x^2 - 4x + 5) = 1$$

find $\delta$ such that

$\boxed{\text{prove}}$ $0 < |x-2| < \delta$ then $|(x^2-4x+5) - 1| < \varepsilon$

(1) Scratch right-side $|(x^2-4x+5)-1| < \varepsilon$

is.. $|x^2-4x+4| < \varepsilon$, so.. $|(x-2)(x-2)| < \varepsilon$

is $|x-2||x-2| < \varepsilon$ compare to left-side $0 < |x-2| < \delta$

let $\delta = \sqrt{\varepsilon}$ ?!

(2) $\boxed{\text{Proof}}$ $0 < |x-2| < \delta$ let $\delta = \sqrt{\varepsilon}$

so.. $|x-2| < \sqrt{\varepsilon}$ then $|x-2||x-2| < \sqrt{\varepsilon}\sqrt{\varepsilon}$

so.. $|(x-2)(x-2)| < \varepsilon$

so.. $|x^2-4x+4| < \varepsilon$

so.. $|x^2-4x+5-1| < \varepsilon$

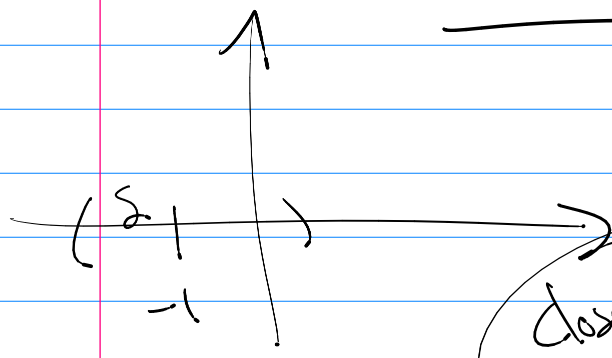
so.. $|(x^2-4x+5)-1| < \varepsilon$

$\therefore$ $|f(x) - L| < \varepsilon$ $\therefore$ done

Note: if you have something like... right-side switch:

$$0 < |x+1| < \delta \to |x+1||\cancel{x-3}| < \varepsilon$$

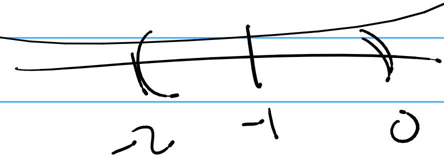
$$|x+1||x-3| < |x+1| \cdot 5$$




$|x+1| < \delta$

close to $-1$

$\delta$ is largest $1$.




So $|x-3|$ @ $x=-2$ is $5$
@ $x=0$ is $3$