Math 242

~~QS~~ Exam #1 Corrections @ 30% missed.

130 → $\boxed{\text{80pts missed}}$

Correct 30% missed

Formulas Solve all problems of a type.

Ex) $ax^2+bx+c=0 \longrightarrow x=\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$ax^2+2bx+c=0 \to x = \dfrac{-b \pm \sqrt{b^2-ac}}{a}$

2.3) Rules/Formulas for derivatives.

$$D_x[f(x)] = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Goal) $D_x[\text{polynomial}]$

$\hookrightarrow a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$

Formulas for...

(1) $D_x[c]$ (4) $D_x[c\,f(x)]$

(2) $D_x[x]$ (5) $D_x[f(x)+g(x)]$

(3) $D_x[x^n]$ $n=2,3,4,\ldots$

① $D_x[\underset{}{\overset{f(x)=c}{c}}] = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \frac{c-c}{h}$

$= \lim\limits_{h\to 0} \frac{0}{h} = \lim\limits_{h\to 0} 0 = 0$

Ex: $D_x[\pi] = 0$

② $D_x[\underset{f(x)=x}{x}] = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \frac{(x+h)-(x)}{h}$

$= \lim\limits_{h\to 0} \frac{h}{h} = \lim\limits_{h\to 0} 1 = 1$

2 Formulas ① $D_x[\text{constant}] = 0$

② $D_x[x] = 1$

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③ $D_x[x^n]$

Note: Binomial th^m

① $n! = n(n-1)(n-2)\cdots(1)$ $\quad 0! = 1$

$3! = 3\cdot 2\cdot 1$

② $(a+b)^n = (a+b)(a+b)\cdots(a+b)$ n-times

$= \overset{=1}{\frac{n!}{n!\,0!}} a^n + \overset{=n}{\frac{n!}{(n-1)!\,1!}} a^{n-1} b^1 + \frac{n!}{(n-2)!\,2!} a^{n-2} b^2 + \cdots + \overset{=1}{\frac{n!}{n!\,0!}} b^n$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$= \underset{\overset{\shortparallel}{1}}{\frac{3!}{3!\,0!}}a^3 + \underset{\overset{\shortparallel}{3}}{\frac{3!}{2!\,1!}}a^2b + \underset{\overset{\shortparallel}{3}}{\frac{3!}{1!\,2!}}ab^2 + \underset{\overset{\shortparallel}{1}}{\frac{3!}{0!\,3!}}b^3$$

$\boxed{\text{So}}$

$$D_x[x^n] = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$$

$f(x) = x^n$

$$= \lim_{h\to 0}\frac{\left(\cancel{x^n} + nx^{n-1}h + \frac{n!}{(n-2)!\,2!}x^{n-2}h^2 + \dots + h^n\right) - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}\left(nx^{n-1} + \frac{n!}{(n-2)!\,2!}x^{n-2}h + \dots + h^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h\to 0} nx^{n-1} + \cancel{\frac{n!}{(n-2)!\,2!}x^{n-2}h} + \cancel{\frac{n!}{(n-3)!\,3!}x^{n-3}h^2} + \dots + \cancel{h^{n-1}}$$

$0 \quad 0 \quad 0$

$$= nx^{n-1}$$

Formula $\quad D_x[x^n] = nx^{n-1}$

ex $\quad D_x[x^4] = 4x^3$

assume $f'(x)$ exists

(4) $D_x[c\,f(x)]$

$$= \lim_{h\to 0} \frac{[c\,f(x+h)] - [c\,f(x)]}{h} = \lim_{h\to 0} c\,\frac{f(x+h)-f(x)}{h}$$

$$= c\left[\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}\right] = c\,f'(x)$$

$$D_x[3x^4] = 3\,D_x[x^4] = 3(4x^3) = \boxed{12x^3}$$

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assume $f'(x), g'(x)$ exist

(5) $D_x[f(x)+g(x)]$

Note

$H(x) = f(x) + g(x)$

$H(x+h) = f(x+h) + g(x+h)$

$H(☺) = f(☺) + g(☺)$

$$= \lim_{h\to 0} \frac{[f(x+h)+g(x+h)] - [f(x)+g(x)]}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h)-f(x)] + [g(x+h)-g(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = f'(x)+g'(x)$$

$\boxed{\text{So}}$ $D_x[f(x) \pm g(x)] = f'(x) \pm g'(x)$ ☆

ex $\frac{d}{dx}\left[3+2x-4x^2+5x^3\right]$

$= (3)' + (2x)' - (4x^2)' + (5x^3)'$

$= 0 + 2\cdot 1 - 4\cdot(2x) + 5\cdot(3x^2)$

$= \boxed{2-8x+15x^2}$

Rational Functions? Products?

$D_x[f(x)g(x)] = ?$

$D_x\left[\frac{f(x)}{g(x)}\right] = ?$

$H(\square) = f(\square)g(\square)$

(6) $D_x[f(x)g(x)]$

$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

remember

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

Idea

If we had

$f(x+h)g(x) - f(x)g(x)$

$(f(x+h) - f(x))g(x)$

So $D_x[f(x)g(x)] = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$

$= \lim_{h\to 0} \left[f(x+h)\left(\frac{g(x+h)-g(x)}{h}\right) + \left(\frac{f(x+h)-f(x)}{h}\right)\cdot g(x) \right]$

$= f(x)g'(x) + f'(x)g(x)$

Product Rule

$$D_x[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

ex

$\frac{d}{dx}\left[(2+x^3)(3-4x^2)\right]$

$= (2+x^3)'(3-4x^2) + (2+x^3)(3-4x^2)'$

$= (3x^2)(3-4x^2) + (2+x^3)(-8x)$

$=$ (finish)

Quotient Rule $D_x\left[\frac{f(x)}{g(x)}\right]$

$$= \lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)\,h}$$

$$= \frac{1}{(g(x))^2} \underbrace{\left[\lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h}\right]}_{\text{do similar to product rule}}$$

$$= \frac{1}{(g(x))^2}\left[f'(x)g(x) - f(x)g'(x)\right]$$

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$D_x\left[x^{-3}\right] = D_x\left[\frac{1}{x^3}\right]$$

$$= \frac{(1)'(x^3) - (1)(3x^2)}{(x^3)^2}$$

$$= \frac{0 - 3x^2}{x^6} = \frac{-3x^2}{x^6} = \frac{-3}{x^4}$$

$$D_x\left[x^{-3}\right] = -3x^{-4}$$

So Power Rule $D_x[x^n] = n x^{n-1}$

$n = \dots, -3, -2, -1, 1, 2, 3, \dots$ (integers)

Note: $\boxed{D_x[x^n] = n x^{n-1}}$ works for all real numbers n.

$$D_x\left[\frac{x^2 - x^{1/2}}{2x^3}\right] = D_x\left[\frac{1}{2}(x^2 - x^{1/2})(x^{-3})\right]$$

Quotient

$$= \frac{(2x - \frac{1}{2}x^{-1/2})2x^3 - (x^2 - x^{1/2})(6x^2)}{(2x^3)^2}$$

Product

$$= \frac{1}{2}\left[(2x - \frac{1}{2}x^{-1/2})(x^{-3}) + (x^2 - x^{1/2})\cdot(-3x^{-4})\right]$$

?

?

Same?

$$\frac{x^2 - x^{1/2}}{2x^3} = \frac{x^2}{2x^3} - \frac{x^{1/2}}{2x^{6/2}}$$

$$= \frac{1}{2}x^{-1} - \frac{1}{2}x^{-5/2}$$

So $\left(\frac{x^2 - x^{1/2}}{2x^3}\right)' = -\frac{1}{2}x^{-2} + \frac{5}{4}x^{-7/2}$