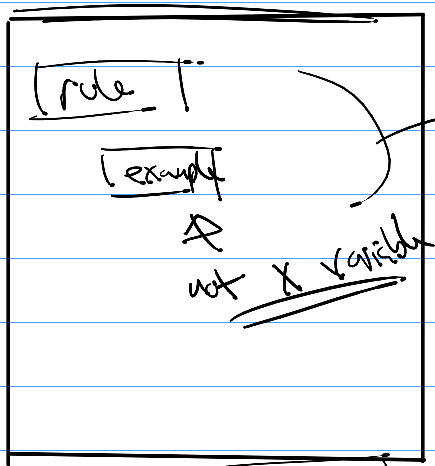
Math 242

~~Q's~~ $\boxed{\text{Exam 2}}$

Note: You $\boxed{\text{Must}}$ (really) know how to take derivatives

Extra Credit (10pts to exam 2) Due Friday

☐ Derivative Sheet (all the rules (+) examples)



ex $\frac{d}{dx}[f\cdot g] = f'g + fg'$

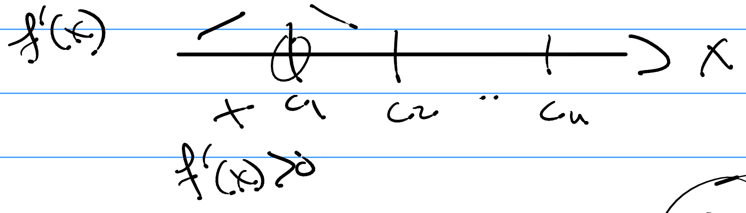
ex $\frac{d}{ds}\left[\sin(s)\cdot(s^{1/3}+2)\right]$

=

(+) $\boxed{\text{2 examples}}$ that use at least 4 rules

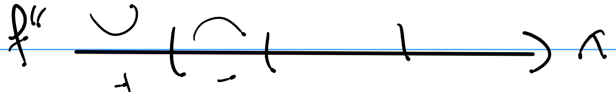
$\boxed{3.3}$ $\boxed{y = f(x)}$ use for domain, intercepts, asymptotes, table of values

slopes $\boxed{f'(x)}$ use inc, dec, extrema (critical numbers), 1st deriv test



concavity $\boxed{f''(x)}$ use concave up, concave down, inflections

2nd deriv. test



ex $f(x)$; find $f'(x)$, find $f''(x)$

$f(x)$

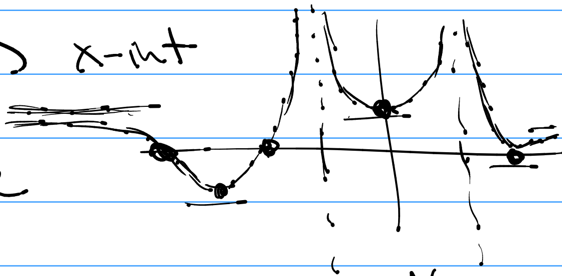
① domain all reals but $x \neq 1$, $x \neq -1$ (vert. asym)

② intercepts were $(0, 2)$ y-int

$(-4, 0)$, $(-2, 0)$, $(3, 0)$ x-int

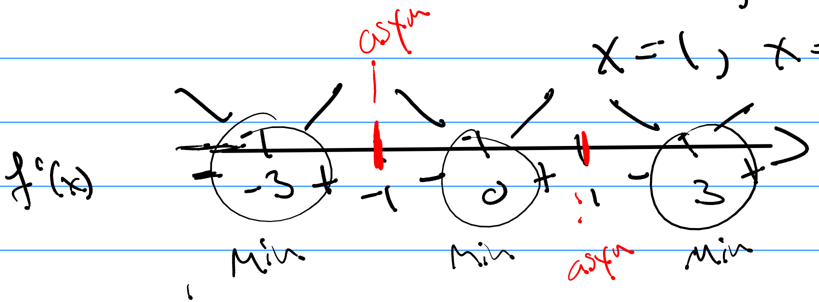
③ right horz asym of $y = 1$

left horz asym of $y = 2$



$f'(x)$ critical numbers of $x = -3$, $x = 3$, $x = 0$ are $f'(x) = 0$

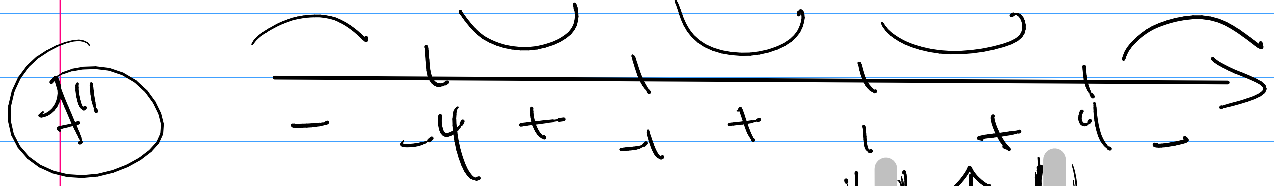
$x = 1$, $x = -$ are $f'(x)$ dne

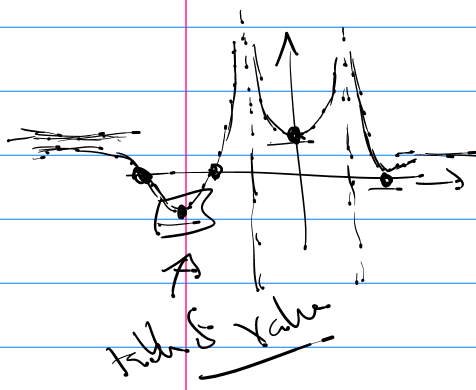


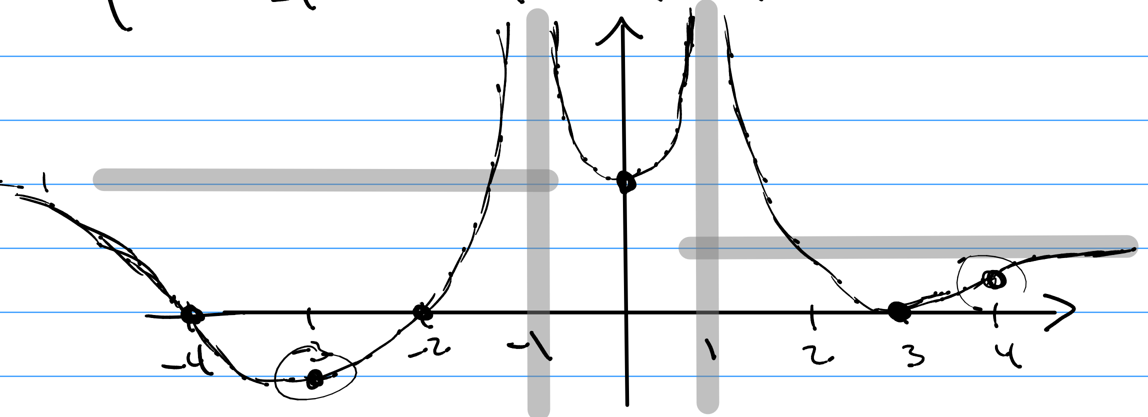
info so far

$f''(x)$ $f''(x) = 0$ @ $x = -3$, $x = 4$

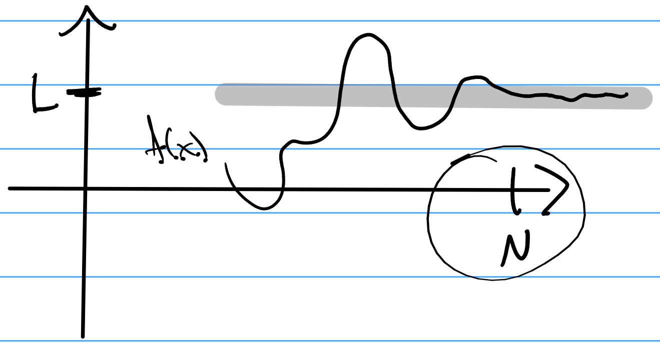
$f''(x)$ dne $x = -1$, $x = 1$

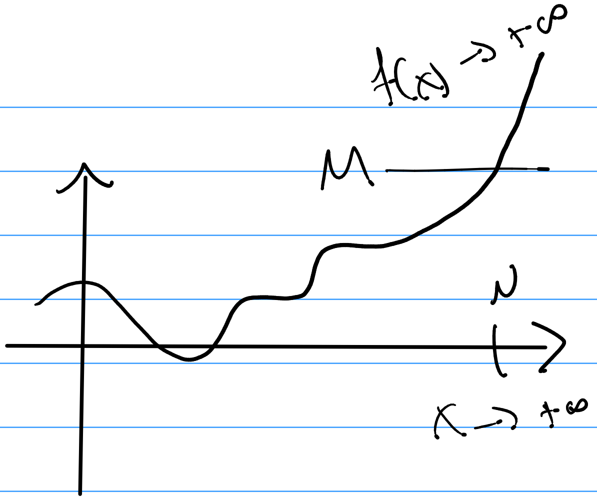






3.4 limits @ infinity





$\lim_{x \to +\infty} f(x) = L$

$\lim_{x \to -\infty} f(x) = L$

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to +\infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

Def: $\lim_{x \to +\infty} f(x) = L$

$\forall \varepsilon > 0 \; \exists N$ if $x > N$, then $|f(x) - L| < \varepsilon$

for all ($\forall$) there exists ($\exists$)

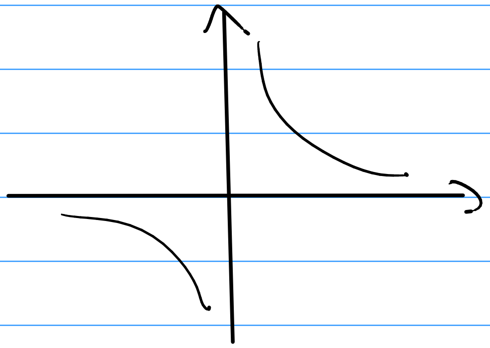
Def $\lim_{x \to -\infty} f(x) = L$

$\forall \varepsilon > 0 \; \exists N$ if $x < N$, then $|f(x) - L| < \varepsilon$

Def: $\lim_{x \to +\infty} f(x) = +\infty$

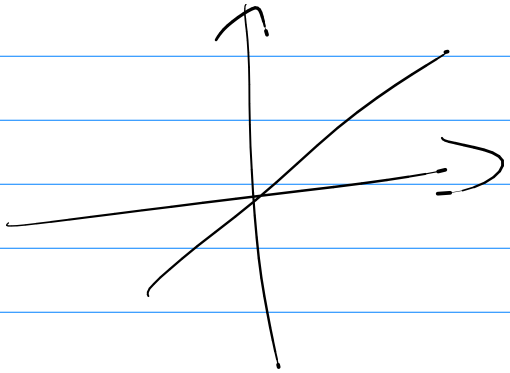
$\forall M \; \exists N$ if $x > N$, $f(x) > M$

$\lim_{x\to+\infty} C = C$, $\lim_{x\to-\infty} C = C$



$\lim_{x\to+\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$



$\lim_{x\to+\infty} x = +\infty$

$\lim_{x\to-\infty} x = -\infty$

→ use limit laws

Note: Thm ① $\lim_{x\to+\infty} \frac{1}{x^r} = \lim_{x\to+\infty} \left(\frac{1}{x}\right)^r = 0$

r is rational

② $\lim_{x\to-\infty} \frac{1}{x^r} = \lim_{x\to-\infty} \left(\frac{1}{x}\right)^r = 0$

if negatives are ok for $\frac{1}{x^r}$

Application: if $\lim_{x\to+\infty} f(x) = L$ call $y=L$ a right horiz. asym

if $\lim_{x\to-\infty} f(x) = L$ call $y=L$ a left horiz. asym ex $f(x) = \frac{x^2-1}{x^2+1}$ $\frac{1}{x}$

$$\lim_{x\to+\infty} \frac{x^2-1}{x^2+1} = \lim_{x\to+\infty} \frac{x^2(1-\frac{1}{x^2})}{x^2(1+\frac{1}{x^2})} = \lim_{x\to+\infty} \frac{1-\frac{1}{x^2}}{1+1/x^2}$$

$$= \frac{1-0}{1+0} = \boxed{1}$$

ex
$$\lim_{x\to-\infty} \frac{x^3-1}{x^2+1} = \lim_{x\to-\infty} \frac{x^3(1-1/x^3)}{x^2(1+1/x^2)}$$

$$= \lim_{x\to-\infty} x \boxed{\left(\frac{1-\overset{0}{\cancel{1/x^3}}}{1+\overset{0}{\cancel{1/x^2}}}\right)} = -\infty(1) = \boxed{-\infty}$$

ex
$$\lim_{x\to-\infty} \left(\overset{+\infty}{\sqrt{4x^2+3x}} - \overset{+\infty}{2x}\right) = +\infty$$

ex
$$\lim_{x\to-\infty} \left(\sqrt{4x^2+3x}+2x\right)\frac{\left(\sqrt{4x^2+3x}-2x\right)}{\left(\sqrt{4x^2+3x}-2x\right)}$$

$$\lim_{x\to-\infty} \frac{(4x^2+3x)-4x^2}{\sqrt{4x^2+3x}-2x} = \lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+3x}-2x}$$

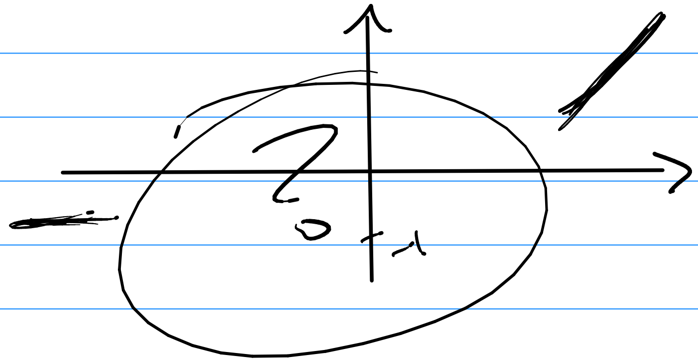
$$= \lim_{x\to-\infty} \frac{x\ (3)}{\underbrace{|x|}_{-x}\sqrt{4+3/x}-2x} = \lim_{x\to-\infty} \frac{\cancel{x}\ (3)}{\cancel{x}\left[-\sqrt{4+3/x}-2\right]}$$

$$= \frac{3}{-\sqrt{4+0}-2} = \boxed{-\frac{3}{4}}$$

So $f(x) = \sqrt{4x^2+3x} + 2x$

left horz. asym of $y = -3/4$

but $\lim_{x\to+\infty} \overset{\to+\infty}{\sqrt{4x^2+3x}} + \overset{\to+\infty}{2x} = +\infty$



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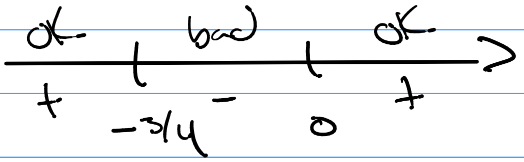
graph?

$f(x) = \sqrt{4x^2+3x} + 2x$

domain: $4x^2+3x \geq 0$ $(-\infty, -3/4] \cup [0, +\infty)$

⊛ $x(4x+3) \geq 0$



intercepts:

① y-int let $x=0 \to y=0$ $(0,0)$

② x-int let $y=0$

$$0 = \sqrt{4x^2+3x} + 2x$$

continued next class