Math 242

Graphing 3.5

$f(x)$: domain, intercepts, table of values, asymptotes
$f'(x)$: criticals number/point, inc/dec, extrema
$f''(x)$: possible inflection points, concave up/down, extrema

simplify → graph

ex $f(x) = \sqrt{4x^2+3x} + 2x$ (continued from last time)

Domain: $(-\infty, -3/4] \cup [0, +\infty)$

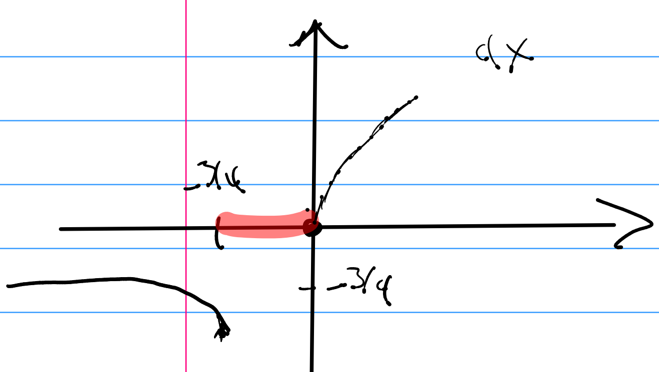
asymptotes: $x \to -\infty$, $f(x) \to -3/4$
$x \to +\infty$, $f(x) \to +\infty$ $f(x) \sim 4x$

Intercepts: y-int (let $x=0$) → point $(0,0)$
x-int (let $y=0$)

Solve $0 = \sqrt{4x^2+3x} + 2x$
$-2x = \sqrt{4x^2+3x}$

$4x^2 = 4x^2+3x \to 3x=0 \to x=0$

Point $(0,0)$



x	$y = \sqrt{4x^3+3x} + 2x$
0	0
-3/4	-3/2

$$f(x) = \sqrt{4x^2+3x} + 2x$$

$$f'(x) = \tfrac{1}{2}(4x^2+3x)^{-1/2}(8x+3) + 2$$

$$f'(x) = \frac{8x+3}{2\sqrt{4x^2+3x}} + 2 = \frac{8x+3+4\sqrt{4x^2+3x}}{2\sqrt{4x^2+3x}}$$

$$f''(x) = \frac{16\sqrt{4x^2+3x} - (8x+3)^2(4x^2+3x)^{-1/2}}{4(4x^2+3x)}$$

$$f''(x) = \frac{16(4x^2+3x) - (8x+3)^2}{4(4x^2+3x)^{3/2}} = \frac{\cancel{64x^2} + \cancel{48x} - \cancel{64x^2} - \cancel{48x} - 9}{4(4x^2+3x)^{3/2}}$$

$$\boxed{f''(x) = \frac{-9}{4(4x^2+3x)^{3/2}}} \overset{\text{always}}{<0}$$ so concave down everywhere!

$$f'(x) = \frac{8x+3+4\sqrt{4x^2+3x}}{2\sqrt{4x^2+3x}}$$

① Criticals $\boxed{f'(x) \overset{\text{never}}{=} 0}$ $\qquad$ $f'(x)$ dne

$$8x+3+4\sqrt{4x^2+3x} = 0$$

$$\sqrt{4x^2+3x} = -2x - 3/4$$

$$4x^2+3x = 4x^2+3x+9/16$$

$$0 = 9/16 \quad \boxed{\text{never}}$$

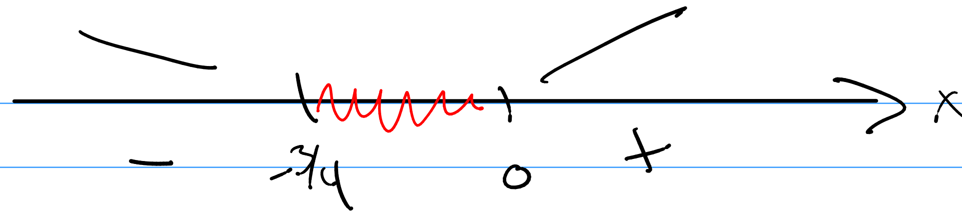
$$4x^2+3x = 0$$

$$x(4x+3) = 0$$

$x=0 \qquad x = -3/4$

$(0,0) \qquad (-3/4, \square)$

$$f'(x) = \frac{8x+3+4\sqrt{4x^2+3x}}{2\sqrt{4x^2+3x}}$$



Ex $f(x) = (x^2-x-6)^3 = (x-3)^3(x+2)^3$

$f'(x) = \boxed{3(x^2-x-6)^2(2x-1)} = \boxed{3(x-3)^2(x+2)^2(2x-1)}$

$f''(x) = [6(x^2-x-6)(2x-1)](2x-1) + 3(x^2-x-6)^2[2]$

$f''(x) = 6(x-3)(x+2)(2x-1)^2 + 6(x-3)^2(x+2)^2$

$f''(x) = 6(x-3)(x+2)[(2x-1)^2 + (x-3)(x+2)]$

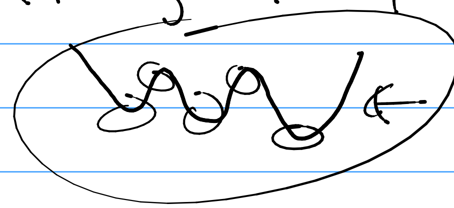
$= 6(x-3)(x+2)[4x^2-4x+1+x^2-x-6]$

$= 6(x-3)(x+2)(5x^2-5x-5)$

$f''(x) = 30(x-3)(x+2)(x^2-x-1)$

$f(x) = (x-3)^3(x+2)^3$ Domain: all reals, no vert asym

x^6



$f'(x) = 3(x-3)^2(x+2)^2(2x-1)$

$f''(x) = 30(x-3)(x+2)(x^2-x-1)$

Intercept: y-int (let $x=0$)

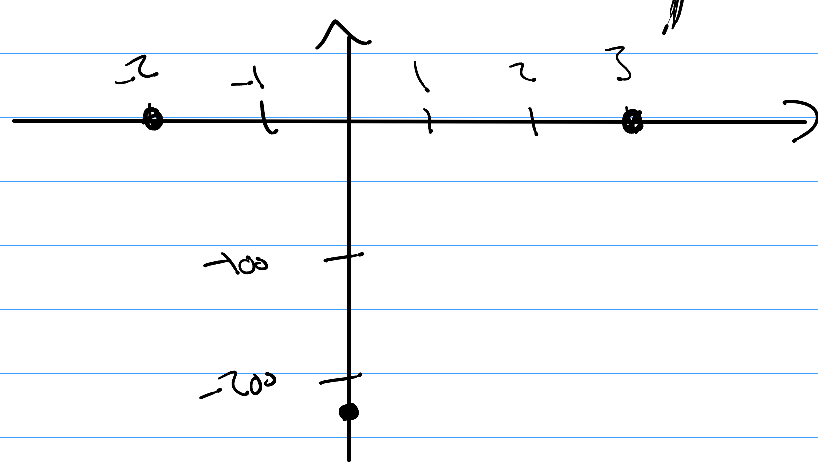
$$f(0) = (0-3)^3(0+2)^3 = (-27)(8) = -216$$

x-int (let $y=0$)

$$0 = (x-3)^3\,(x+2)^3$$

$x = 3$ $\quad$ $x = -2$

$(3,0)$ $\quad$ $(-2,0)$



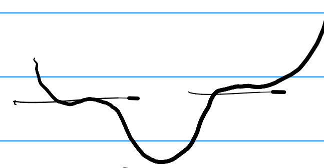
Use $f'(x) = 3(x-3)^2(x+2)^2(2x-1)$

① **Criticals** $f'=0$ $\qquad$ f' dne **never**

$$3(x-3)^2(x+2)^2(2x-1)=0$$

$+ \qquad + \qquad +$

$x=3$, $x=-2$, $x=1/2$



② f': $\quad$ $-2 \quad 1/2 \quad 3$

$- \quad - \quad + \quad +$

not an extrema $\quad$ rel. min $\quad$ not an extrema

Ex $f''(x) = 30(x-3)(x+2)(x^2-x-1)$

① possible inflections

$f''(x) = 0$

$x=3$, $x=-2$

$x^2-x-1=0$

$x = \frac{1 \pm \sqrt{1+4}}{2}$

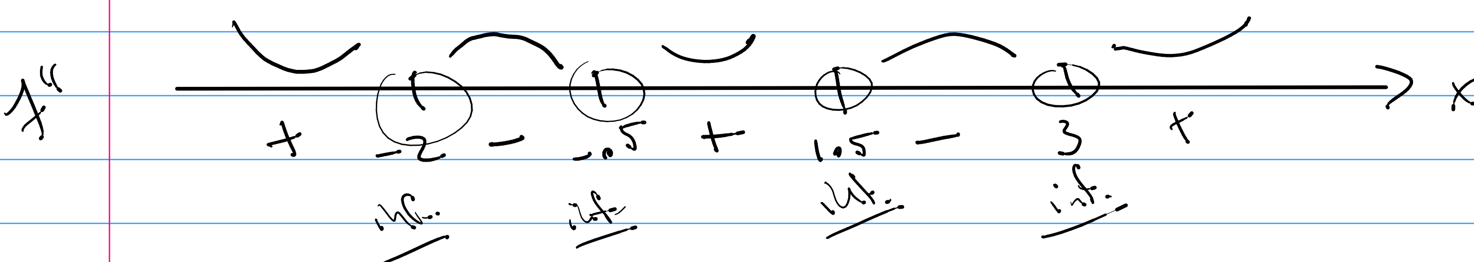
$x = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$ $\approx -.5$, $1.5 \approx$

$f''(x)$ dne

never

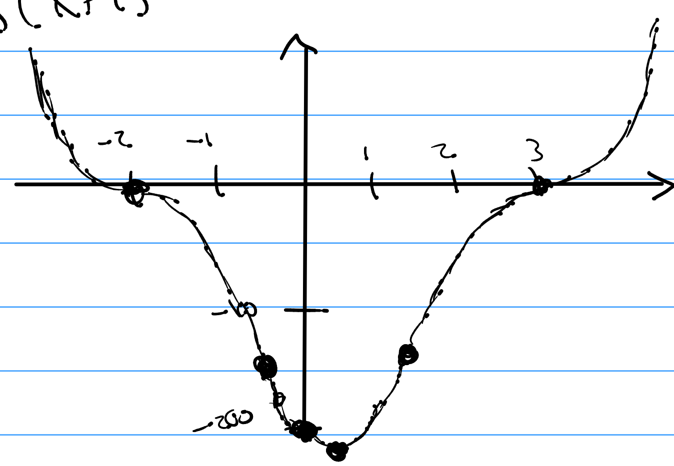
②

$f''(x) = 30(x-3)(x+2)(x^2-x-1)$



table

x	$y = (x-3)^3(x+2)^3$
-2	0
$(1-\sqrt{5})/2$	
0	-216
$1/2$	
$(1+\sqrt{5})/2$	
3	0



$y = \left(\frac{1+\sqrt{5}}{2} - 3\right)^3 \left(\frac{1+\sqrt{5}}{2} + 2\right)^3 = ?$