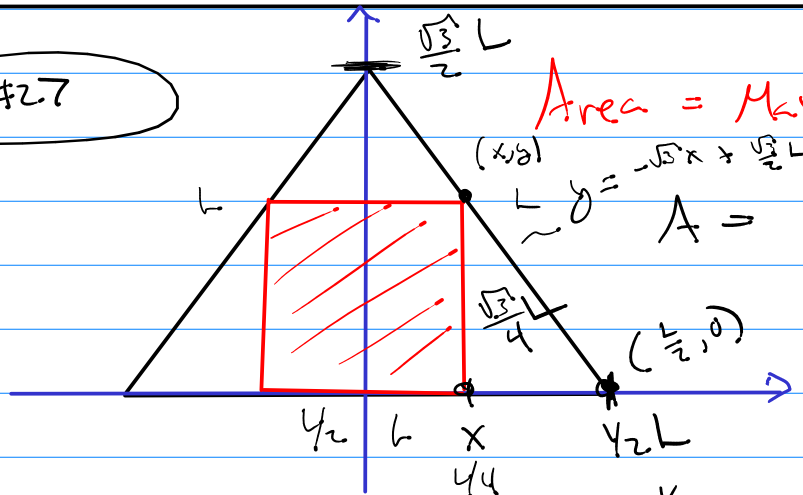
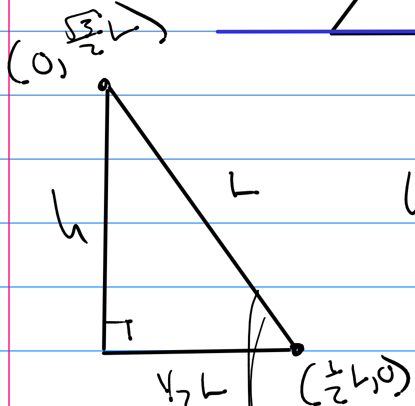
Math 242

Q5 / 3.7 #27



$A = (2x)(y) = 2xy$

need one variable to optimize



$$h = \left(L^2 - \left(\tfrac{1}{2}L\right)^2\right)^{1/2}$$

$$h = \left(\tfrac{3}{4}L^2\right)^{1/2} = \frac{\sqrt{3}}{2}L$$

$$m = \frac{\Delta y}{\Delta x} = -\frac{\frac{\sqrt{3}}{2}L}{\frac{1}{2}L} = -\sqrt{3}$$

$$y = -\sqrt{3}x + \frac{\sqrt{3}}{2}L$$

$A = 2xy$, $y = -\sqrt{3}x + \frac{\sqrt{3}}{2}L$

So $A(x) = 2x\left(-\sqrt{3}x + \frac{\sqrt{3}}{2}L\right)$

$A(x) = -2\sqrt{3}x^2 + \sqrt{3}Lx$ Domain: $[0, L/2]$

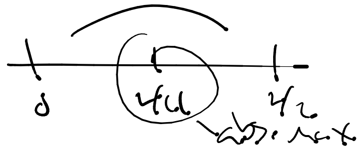
critical numbers: $A'(x) = 0$ or $A'(x)$ dne

$-4\sqrt{3}x + \sqrt{3}L = 0$ or never

$x = L/4$

$A'(x)$: 0 $+$ $L/4$ $-$ $L/2$ — rel max → abs max @ $L/4$

$$A''(x) = -4\sqrt{3}$$



abs max of $A(L/4) = \boxed{\frac{\sqrt{3}}{8} L^2}$

$$A(x) = -\sqrt{3}\frac{L^2}{8} + 2\sqrt{3}\frac{L^2}{8} = \boxed{\frac{\sqrt{3}}{8} L^2}$$

3.9 Anti Derivatives

① $A_x[\cos x] = \sin x + C$

b/c $D_x[\sin x + C] = \cos x$

② $A_x[\sin x] = -\cos x + C$

b/c $D_x[-\cos x + C] = \sin x$

③ $A_x[\sec^2 x] = \tan x + C$

b/c $D_x\{\tan x + C\} = \sec^2 x$

④ $A_x[\csc^2 x] = -\cot x + C$

b/c $D_x[-\cot x + C] = \csc^2 x$

⑤ $A_x[\sec x \tan x] = \sec x + C$

b/c $D_x[\sec x + C] = \sec x \tan x$

⑥ $A_x[\csc x \cot x] = -\csc x + C$

$D_x[-\csc x + C] = \csc x \cot x$

⑦ $A_x[x^n] = \frac{1}{n+1} x^{n+1} + C$

$(n \neq -1)$

$D_x\left[\frac{1}{n+1} x^{n+1} + C\right] = x^n$

⑧ $A_x[f(x)+g(x)] = A_x[f(x)] + A_x[g(x)]$

⑨ $A_x[k\,f(x)] = k\,A_x[f(x)]$

(ex) $A_x[4x+3x^2-2]$

$= 4A_x[x] + 3A_x[x^2] - 2A_x[x^0]$

$= 4\frac{1}{2}x^2 + 3\frac{1}{3}x^3 - 2x + c$

(ex) $A_t[8\sqrt{t} - \sec t \tan t]$

$= A_t[8t^{1/2} - \sec t \tan t] = 8\cdot\frac{2}{3}t^{3/2} - \sec t + C$

$= \frac{16}{3}t^{3/2} - \sec t + c = \boxed{\frac{16}{3}(\sqrt{t})^3 - \sec t + c}$

D_t[position function] = velocity function

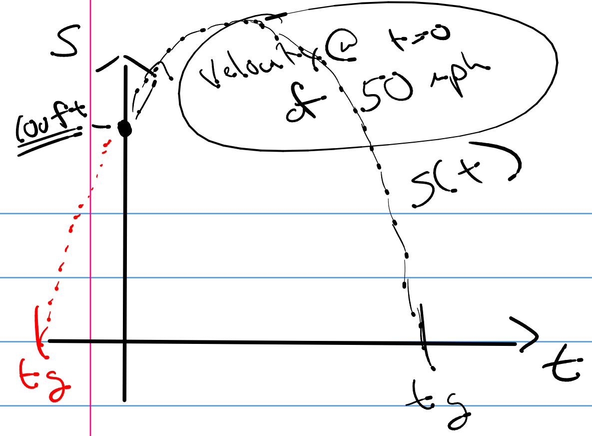
D_t[velocity function] = acceleration function

Says ① A_t[acceleration] = velocity + C_1

find this by 1 velocity measurement

② A_t[velocity] = position + C_2

find this by 1 position measurement



ignore air resistance

accel of gravity

gravity ~ $-32 \frac{ft/sec}{sec}$

Force = gravity = M · accel.

$a(t) = -32$

$v(t) + C = A_t[\text{accel}] = A_t[-32]$

$v(t) = -32t + c_1$

$t=0 \quad V = 50 \frac{\text{miles}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}}$

$V \approx 73$ ft/sec

$$\boxed{v(t) = -32t + 73}$$ ← ft/sec

Now position $= A_t[\text{velocity}]$

$s(t) = A_t[-32t + 73] = -16t^2 + 73t + C_2$

@ $t=0 \quad s = 100$ ft

$$\boxed{s(t) = -16t^2 + 73t + 100}$$

when is $S=0$? (on ground)

$0 = -16t^2 + 73t + 100$

$$t = \frac{-73 \pm \sqrt{73^2 + 4 \cdot 16 \cdot 100}}{-32}$$

3.9 #62

@ $t=0$ $V_0 = 48$ ft/sec

$t=1$ $V_0 = 24$ ft/sec

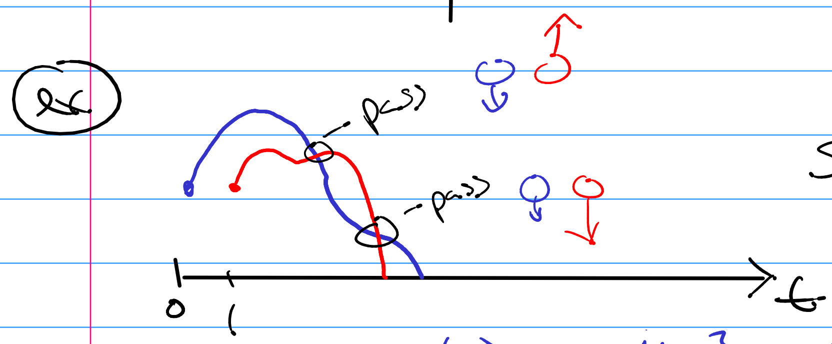
$V_0 \equiv$ initial velocity.

Do the balls pass each other?

Same height @ same time?

velocity?

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$, $s_0 = 0$



$S_b(t) = -16t^2 + 48t$

$S_r(t) = -16(t-1)^2 + 24(t-1)$

$+32t - 16 + 24t - 24$

$S_b(t) = -16t^2 + 48t$

$S_r(t) = -16t^2 + 56t - 40$

$t \geq 1$

$-16t^2 + 48t = -16t^2 + 56t - 40$

$8t = 40$

$t = 5$ sec