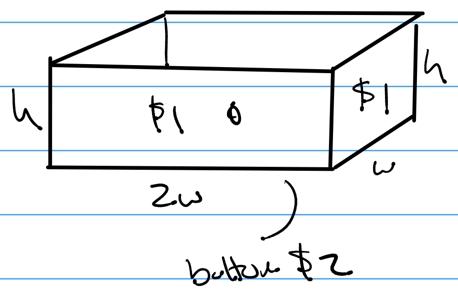
Math 242

Q's exam 3 #14



$8 = 2w^2h \rightarrow h = \frac{4}{w^2}$

$Cost = \$2(2w^2) + \$1(4wh + 2wh)$

$Cost = 4w^2 + 6wh$

$\boxed{C(w) = 4w^2 + \frac{24}{w}}$ domain: $(0, +\infty)$

extrema: $C'(w)$ etc.

$(3-x)\sqrt{x} = 0$

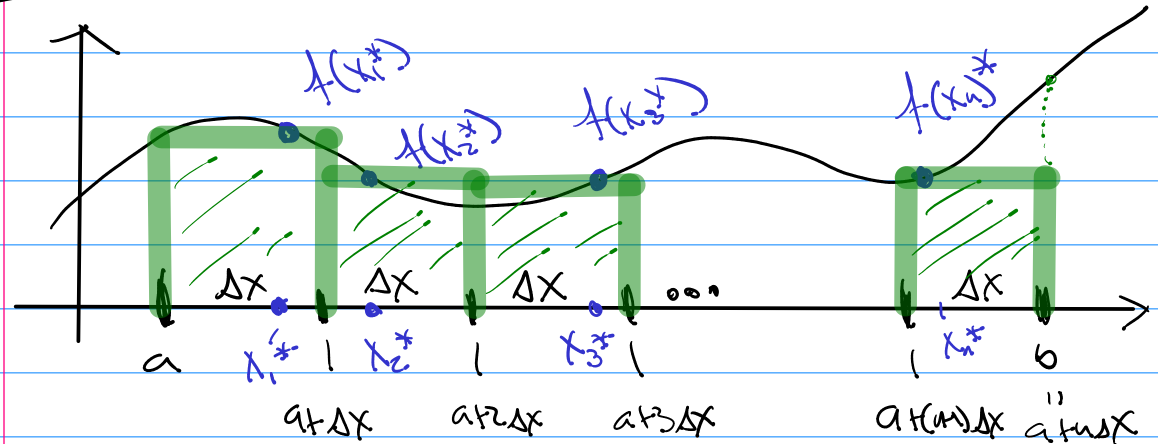
$x=3 \quad x=0$

$3x^{1/2} - x^{3/2} = f(x)$

$f'(x) = \frac{3}{2}x^{-1/2} - \frac{3}{2}x^{1/2}$

Integral Calculus

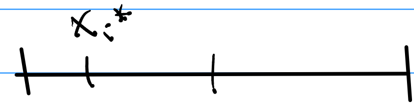
n-intervals: $\Delta x = \frac{b-a}{n}$



$Area \approx f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x$

$Area \approx \left(\frac{f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)}{n}\right)(b-a)$

x_i^* are $\boxed{\text{any}}$ value in their interval.



$\boxed{a+(i-1)\Delta x}$ $\boxed{a+i\Delta x}$

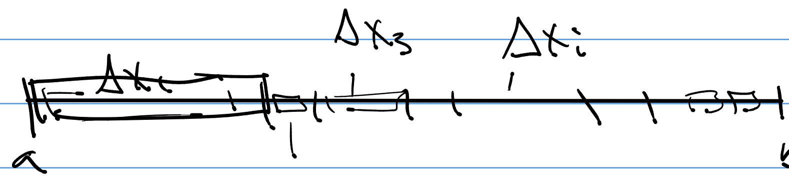
left end point approx : x_i^* = left end point

right end point approx : x_i^* = right end point

mid point approx ; $x_i^* = a + (i - \frac{1}{2})\Delta x$

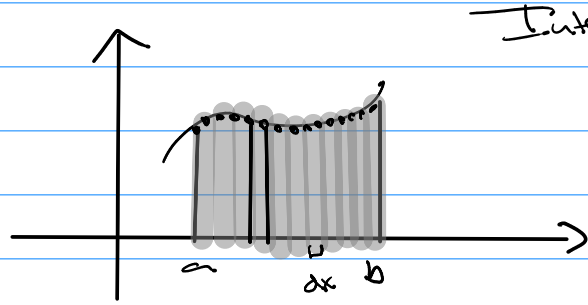
$$\text{Area} = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

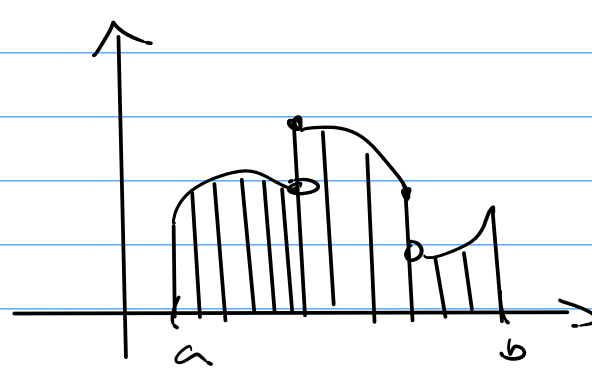
$$\boxed{\text{Area} = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i}$$



Leibniz Notation $\boxed{\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x}$

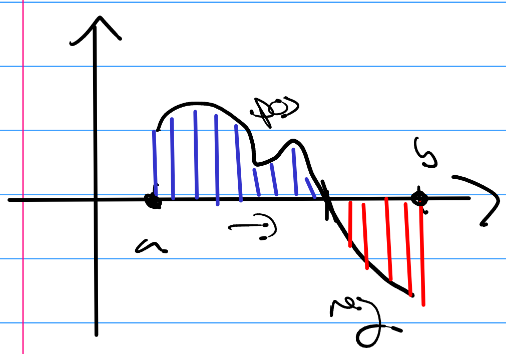
Integral , Definite Integral





Thm if f is continous on $[a,b]$ <u>or</u> has at most a finite number of jump discontinuites

(Integrable) then $\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$ <u>exists</u>



$\int_a^b f(x)\,dx$ = net signed area

or

net area

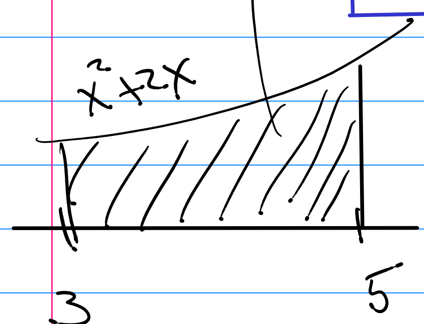
Thm if f is integrable then $\Delta = \frac{b-a}{n}$

$$\int_a^b f(x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

Notation:

$$= \boxed{\int_3^5 (x^2+2x)\,dx} = \lim_{n\to+\infty} \sum_{i=1}^{n} (x_i^2+2x_i)\,\Delta x$$

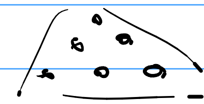
$$\Delta x = \frac{5-3}{n}$$



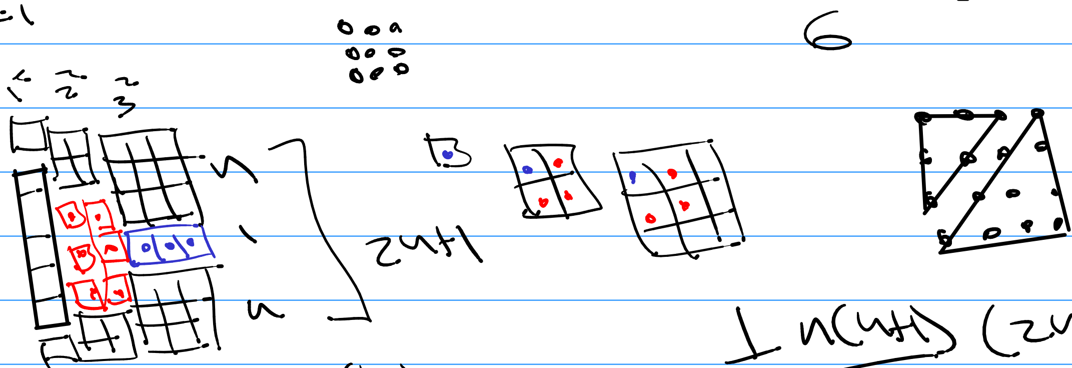
Sums to Know

① $\sum_{i=1}^{n} c = \underbrace{c+c+c+c+\dots+c}_{n} = c \cdot n$

② $\sum_{i=1}^{n} i = \boxed{1+2+3+\dots+n} = \frac{n(n+1)}{2}$



③ $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

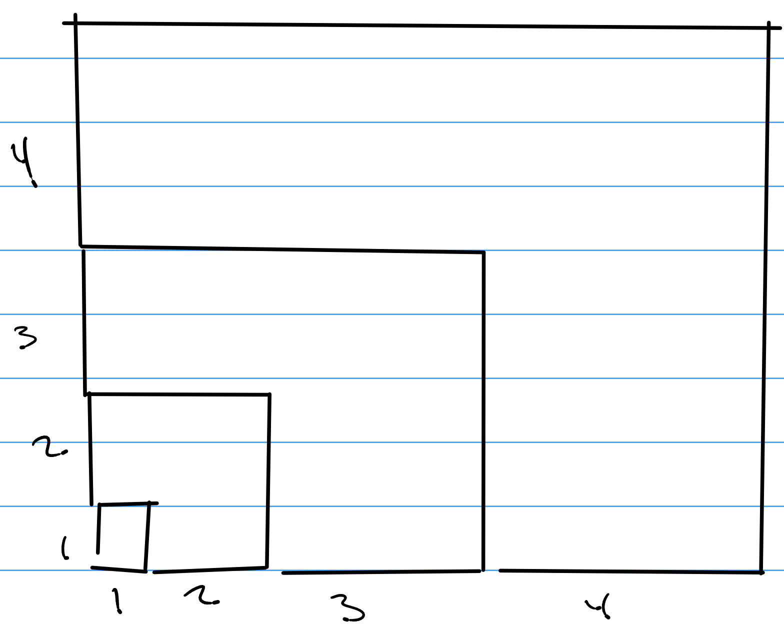


$1+2+3+\dots = \frac{n(n+1)}{2}$

$\frac{1}{3} \frac{n(n+1)}{2}(2n+1)$

④ $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1^3+2^3+3^3+4^3 = \left(\frac{4(4+1)}{2}\right)^2$



$$\sum_{i=1}^{n} c a_i = c \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} a_i \pm b_i = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

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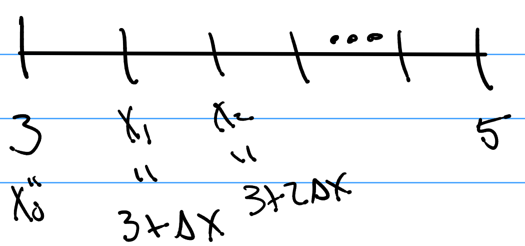
ex $\displaystyle\int_3^5 (x^2+2x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} (x_i^2+2x_i)\Delta x$

$$\Delta x = \frac{5-3}{n} = \frac{2}{n}$$



right endpts

$$x_i = 3 + i\Delta x = 3 + \frac{2i}{n}$$

$$\int_3^5 (x^2+2x)\,dx = \lim_{n\to+\infty} \frac{2}{n} \sum_{i=1}^{n} \left(3+\frac{2}{n}i\right)^2 + 2\left(3+\frac{2}{n}i\right)$$

$$= \lim_{n\to+\infty} \frac{2}{n} \sum_{i=1}^{n} \left(9 + \frac{12}{n}i + \frac{4}{n^2}i^2\right) + \left(6 + \frac{4}{n}i\right)$$

$$= \lim_{n\to+\infty} \frac{2}{n} \sum_{i=1}^{n} \left(15 + \frac{16}{n}i + \frac{4}{n^2}i^2\right)$$

$$= \lim_{n\to+\infty} \left(\frac{2}{n}\left[15n + \frac{\cancel{16}^{8}}{n}\frac{n(n+1)}{\cancel{2}} + \frac{\cancel{4}^{2}}{n^2}\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right]\right)$$

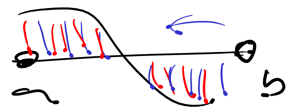
$$= \lim_{n\to+\infty} \left(30 + 16\cdot\frac{n(n+1)}{n^2} + \frac{4}{3}\frac{n(n+1)(2n+1)}{n^3}\right)$$

$$= 30 + 16\cdot 1 + \frac{4}{3}\cdot 2$$

$$= 46 + \frac{8}{3} = \boxed{48 + 2/3} = 48.\overline{6}$$

Properties

$$\int_a^b f(x)\,dx$$



① $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

② $\int_a^a f(x)\,dx = 0$

③ $\int_a^b c\,dx = c(b-a)$

④ $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑤ $\int_a^b (c\,f(x))\,dx = c\int_a^b f(x)\,dx$

⑥ $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$