Math 242

~~Q's~~ $\int_{-a}^{a}(ax^4+bx+c)\,dx \overset{?}{=} 2\int_0^a (ax^4+c)\,dx$

Know

$$\int_{-a}^{a}(\text{even function})\,dx = 2\int_0^a(\text{even function})\,dx$$

$$\int_{-a}^{a}(\text{odd function})\,dx = 0$$

$$\to 2\int_0^a ax^4\,dx + \cancel{\int_{-a}^{a} bx\,dx} + 2\int_0^a cx^0\,dx$$

Q's $\int_{-3}^{3}\left(\boxed{\cancel{7x^5}} - 2x^4 + 3x^2 \boxed{\cancel{-x}}\ \boxed{\cancel{+\sin x}} + 1\right)dx$

$$= \int_{-3}^{3}(-2x^4+3x^2+1)\,dx = \text{etc.}$$

Know ① $\int f(x)\,dx = F(x) + C$

$F(x)$ = antideriv. of $f(x)$

② $\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]\Big|_{x=a}^{x=b} = F(b) - F(a)$

③ Antiderivative?

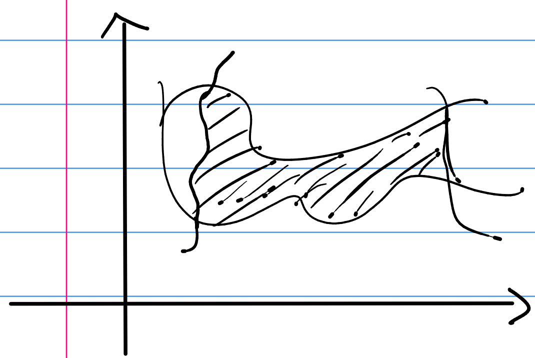
a) table?

b) Algebra / trig ?

c) Substitution Method?

Ch 5 Applications for integration

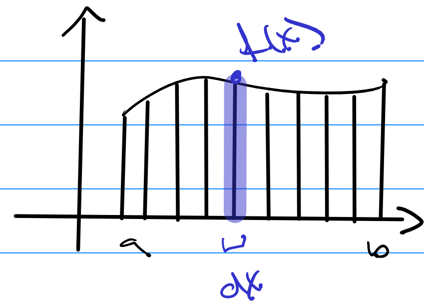
5.1 Area between curves (always positive)



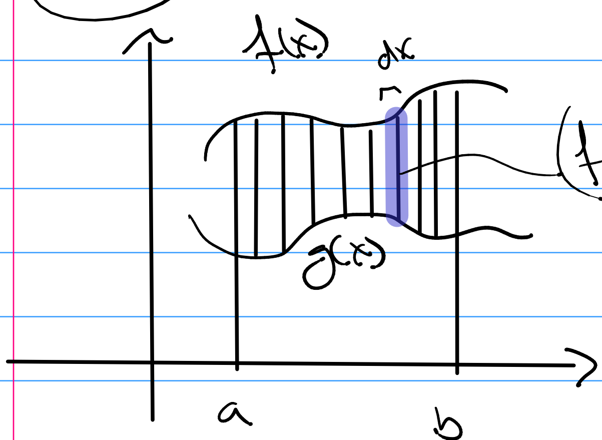
Compare to

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

$f(x) \geq 0$

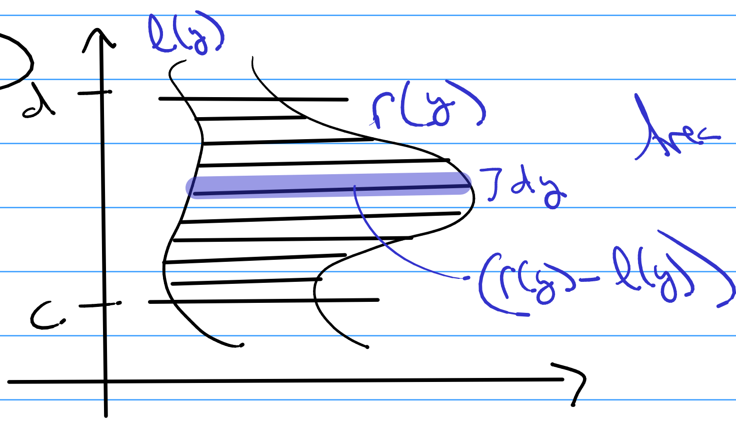


vs $f(x) \geq g(x)$



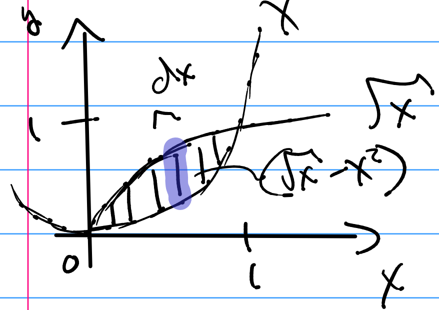
$$\int_a^b (f(x) - g(x))\,dx$$

vs



$$\text{Area} = \int_c^d (r(y) - l(y))\,dy$$

ex Area between $\sqrt{x}$ and x^2?



Cross?

$$\sqrt{x} = x^2$$
$$x = x^4$$
$$x^4 - x = 0$$
$$x(x^3-1) = 0$$
$$x=0 \quad x=1$$

$$\int_0^1 (\sqrt{x} - x^2)\,dx = \frac{2}{3}x^{3/2} - \frac{1}{3}x^3 \Big|_{x=0}^{x=1}$$
$$= \left(\frac{2}{3} - \frac{1}{3}\right) - (0) = \boxed{\frac{1}{3}}$$

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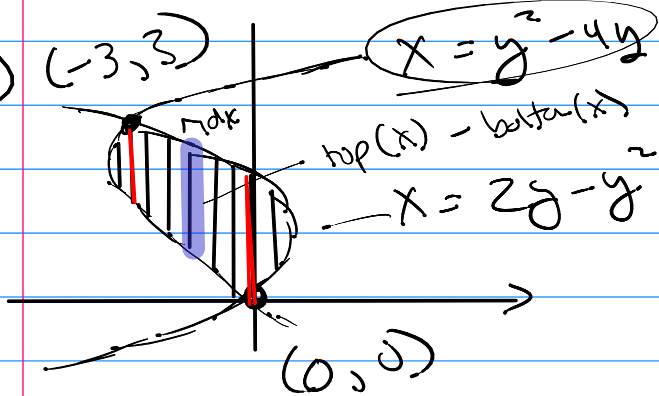
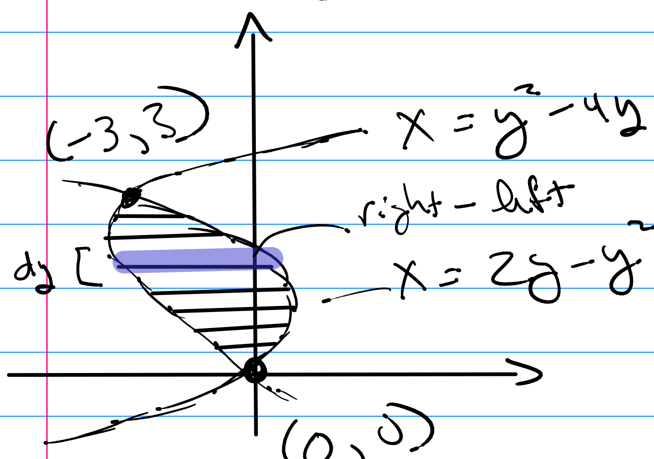
ex $x = y^2 - 4y$



Cross?

$$y^2 - 4y = 2y - y^2$$
$$2y^2 - 6y = 0$$
$$2y(y-3) = 0$$
$$y=0 \quad y=3$$



$$\int_0^3 \left((2y-y^2) - (y^2-4y)\right)dy$$

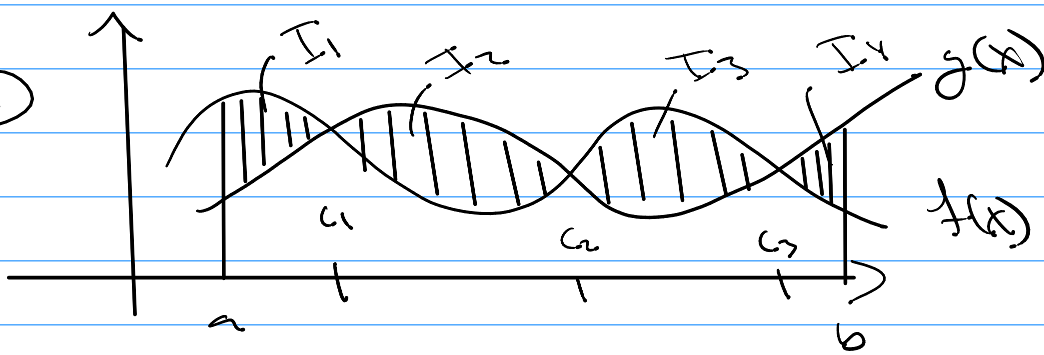
Do this!

$$\text{Area} = \int_0^3 (-2y^2 + 6y)\,dy = \left(-\frac{2}{3}y^3 + 3y^2\right)\Big|_{y=0}^{y=3}$$
$$= \left(-\frac{2}{3}(3)^3 + 3(3)^2\right) - (0) = \boxed{9}$$

① $f(x) \geq g(x)$ Area $= \int_a^b \left(f(x) - g(x)\right) dx$

② don't know? Area $= \int_a^b \left| f(x) - g(x) \right| dx$

ex



Area $= \int_a^b \left| f(x) - g(x) \right| dx$

Area $= \int_a^{c_1} (f - g)\,dx + \int_{c_1}^{c_2} (g - f)\,dx + \int_{c_2}^{c_3} (f - g)\,dx + \int_{c_3}^{b} (g - f)\,dx$

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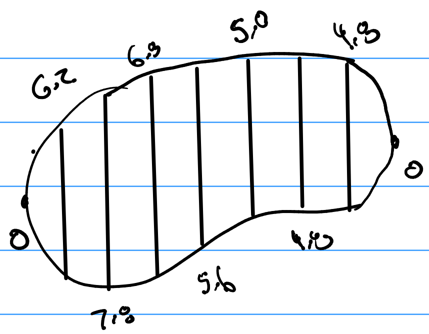
ex



$\Delta x = 2$

Area ≈ ① left end pt

② right end pt

③ mid point

length: 0, 6.2, 7.8, 6.8, 5.6, 5.0, 4.8, 4.8, 0

left end pt Area $\approx \left(\dfrac{0 + 6.2 + 7.8 + 6.8 + 5.6 + 5.0 + 4.8 + 4.8}{8} \right) 16$

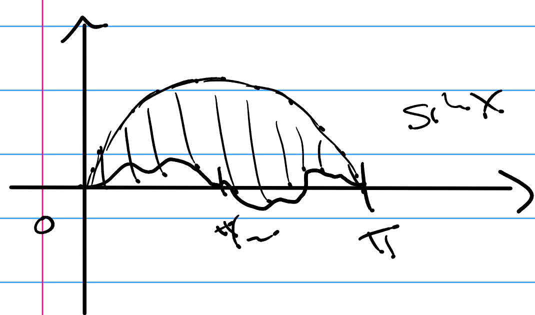
midpoints Area $\approx \left(\dfrac{6.2 + 6.8 + 5.0 + 4.8}{4} \right) 16$

Simpson's

1	4	2	4	2	4	2	4	1
0	6.2	7.8	6.8	5.6	5.0	4.8	4.8	0

$$\text{Area} \approx \left(\frac{0 + 4\cdot 6.2 + 2\cdot 7.8 + 4\cdot 6.8 + 2\cdot 5.6 + 4\cdot 5.0 + 2\cdot 4.8 + 4\cdot 4.8 + 0}{24}\right) 16$$

Ex $y = \cos^2 x \sin x$, $y = \sin x$, between $0, \pi$



cross?

$$\cos^2 x \sin x = \sin x$$

$$\sin x(\cos^2 x - 1) = 0$$

$$\sin x = 0 \qquad \cos^2 x = 1$$

$$0, \pi \qquad \cos x = 1 \quad \cos x = -1$$

$$x = 0, 2\pi \qquad \pi$$

$$\text{Area} = \int_0^{\pi} (\sin x - \cos^2 x \sin x)\,dx$$

$$= \int_0^{\pi} \sin x\,dx - \int_0^{\pi} (\cos x)^2 \sin x\,dx$$

$$= (-\cos x)\Big|_{x=0}^{x=\pi} - \int_0^{\pi} (\cos x)^2 \sin x\,dx$$

$$= \left((-(-1)) - (-1)\right) - \int_0^{\pi} (\cos x)^2 \sin x\,dx$$

$$= 2 - \int_0^{\pi} (\cos x)^2 \sin x\,dx$$

let $u = \cos x$ $\qquad x = 0 \to u = 1$

$du = -\sin x\,dx$ $\qquad x = \pi \to u = -1$

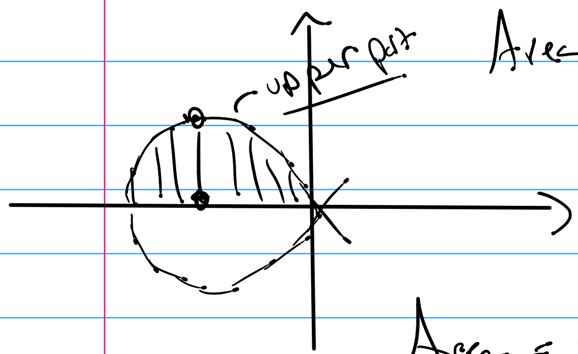
$$= 2 + \int_{u=1}^{u=-1} u^2\,du = 2 + \frac{1}{3}u^3\Big|_{u=1}^{u=-1}$$

$$= 2 + \left(\left(-\frac{1}{3}\right) - \left(\frac{1}{3}\right)\right) = 2 - \frac{2}{3} = \boxed{\frac{4}{3}}$$

Ex $y^2 = x^2(x+3) \to y = +\sqrt{x^2(x+3)}$

$\searrow y = -\sqrt{x^2(x+3)}$

if $y = 0 \to x = 0,\ x = -3$



Area of loop $= 2\int_{-3}^{0} (\text{upper part})\,dx$

$$\text{Area} = 2\int_{-3}^{0} \sqrt{x^2(x+3)}\,dx = 2\int_{-3}^{0} \underbrace{|x|}_{=-x \text{ in } [-3,0]} \sqrt{x+3}\,dx$$

$$\text{Area} = -2\int_{-3}^{0} x\sqrt{x+3}\,dx = \text{use substitution}$$