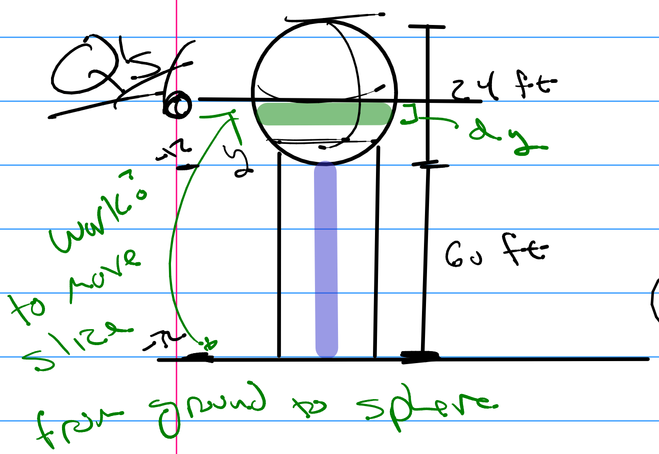
Math 242

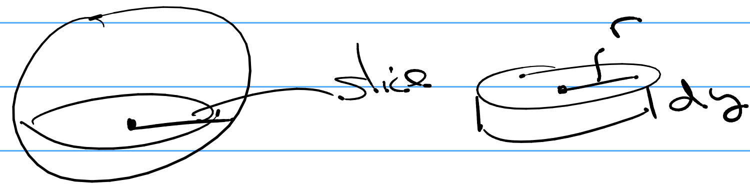


pump $= 1.5$ HP $= 1.5(550 \text{ ft·lb})/\text{sec}$

$= 825 \frac{\text{ft·lb}}{\text{sec}}$ — $\frac{\text{work}}{\text{sec}}$

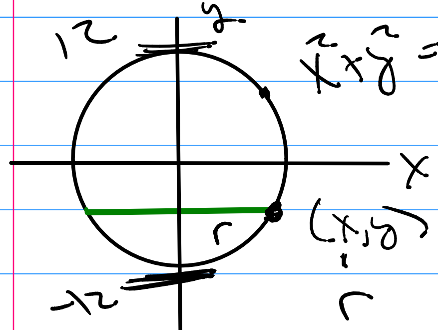
$\frac{\text{Total Work}}{825 \text{ work/sec}} =$ total time sec

$\rho = 62.5 \text{ lb/ft}^3$



Volume $= \pi r^2 dy$

weight $= \pi r^2 dy \cdot \rho$



$x^2 + y^2 = 144$

$r = f(y)$

$r^2 + y^2 = 144$

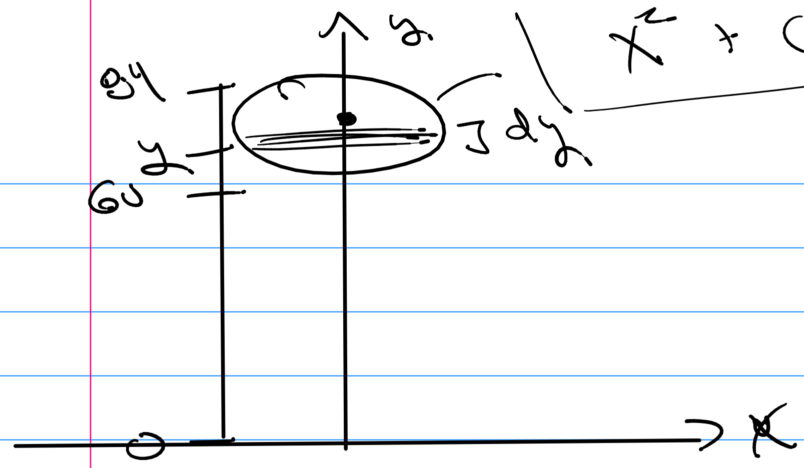
$r^2 = 144 - y^2$

<u>bottom of tank</u>: @ $y = -12$, weight $= \pi(144 - y^2)dy \cdot (62.5)$ lb

displacement $= 72 + (-12) = 60$

$\text{Work}_{\text{bottom}} = \text{weight} \cdot \text{displacement}$

$$\text{Work} = \int_{-12}^{12} \left(\pi(144 - y^2)\, 62.5\, dy\right)(72 + y)$$



$x^2 + (y-k)^2 = r^2$

(y)

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ch6 Inverse Functions

horiz. line test

Existence if f is one-to-one and onto (bijection)

then f^{-1} exists

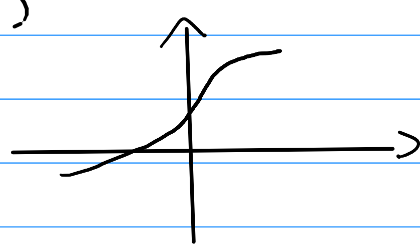
Find f^{-1}

① $y = f(x)$

② interchange x, y.

③ $y = f^{-1}(x)$

ex $\boxed{f(x) = x^{1/3} + 1}$



① see that f^{-1} must exist

② $x = y^{1/3} + 1 \xrightarrow{\text{algebra}} y = (x-1)^3$

③ $\boxed{f^{-1}(x) = (x-1)^3}$

$f(f^{-1}(x)) = ((x-1)^3)^{1/3} + 1 = x$

$f^{-1}(f(x)) = ((x^{1/3}+1) - 1)^3 = x$

Ex $f(x) = x^{1/3} + x$

① passes horz. line test (one-to-one)
So f^{-1} exists

② $y = x^{1/3} + x \xrightarrow[f^{-1}]{\text{find}} \boxed{x = y^{1/3} + y}$

$\underbrace{}_{\cancel{\text{algebra}}}$

$y = f^{-1}(x)$ ←

$y = f^{-1}(x)$ exists
but I can not write it explicitly.

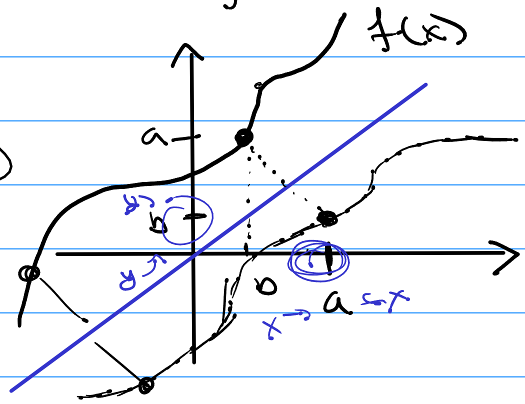
Transcendental Functions

are functions that can not be written with a finite number of algebraic operations.
+, ×, power, roots

Derivatives of f^{-1}

Thm f is one-to-one (f^{-1} exists) and continuous, then f^{-1} is also continuous.

graphs



$f(b) = a$ iff $f^{-1}(a) = b$

$f(x) = y$ iff $f^{-1}(y) = x$

$f(\square) = \Delta$ iff $f^{-1}(\Delta) = \square$

$D_x[f^{-1}(x)]$

slope of $f^{-1}(x)$ @ $x=a$ $\quad \boxed{(f^{-1})'(a)}$

$$(f^{-1})'(a) = \lim_{x\to a} \frac{f^{-1}(x) - f^{-1}(a)}{x-a}$$

$\boxed{\text{Know}}$ $\quad f^{-1}(x) = y$ iff $f(y) = x$ $\quad$ as $x \to a$

$f^{-1}(a) = b$ iff $f(b) = a$ $\quad y \to b$

$$(f^{-1})'(a) = \lim_{y\to b} \frac{y-b}{f(y)-f(b)} = \frac{1}{\lim_{y\to b} \frac{f(y)-f(b)}{y-b}}$$

$$= \frac{1}{f'(b)} = \frac{1}{f'(f^{-1}(a))}$$

Different Notation ① slope @ $x=a$ of f^{-1}

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

② $$D_x[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$$

Ex $\quad f(x) = x^{1/3}+1 \qquad f^{-1}(x) = (x-1)^3$

obvious: $D_x[f^{-1}(x)] = D_x[(x-1)^3] = 3(x-1)^2$

$\boxed{\text{also}}$ $\quad f'(x) = \frac{1}{3}x^{-2/3}$

$$D_x[f^{-1}(x)] = \frac{1}{\frac{1}{3}((x-1)^3)^{-2/3}} = 3(x-1)^2$$

6.2 A new transcendental function

(exponential $f(x) = b^x$)

① why? geometric growth...

a) $1,000 0

b) $1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + \dots + 2^{60} = 2^{61} - 1$

Compounding interest $A = P\left(1 + \frac{r}{n}\right)^{nt}$

cont. compounding $n \to +\infty$

$$A = P \lim_{n \to +\infty} \left(1 + \frac{r}{n}\right)^{nt}$$

let $h = \frac{n}{r}$

$$A = P\left[\underbrace{\lim_{h \to +\infty} \left(1 + \frac{1}{h}\right)^h}_{\text{const.} = e}\right]^{rt}$$

$$\boxed{A = Pe^{rt}}$$

$f(x) = b^x$ need $b > 0$

Properties

① $b^n = \underbrace{b \cdot b \cdots b}_{n \text{ factors}}$ ← n pos. int

② $b^{-n} = \dfrac{1}{b^n} = \dfrac{1}{b} \cdot \dfrac{1}{b} \cdots \dfrac{1}{b}$

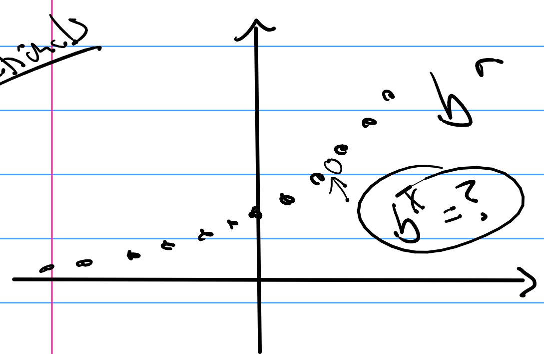
③ b^r where $r = \dfrac{p}{q}$ a rational number

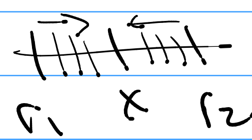
$$b^{p/q} = \left(b^{1/q}\right)^p$$

where $b^{1/q} = a$ when $a^q = b$

④ $x \in \mathbb{R}$

rationals





$$b^x = \lim_{r \to x} b^r$$

⑤ $b^x b^y = b^{x+y}$ ⑦ $(b^x)^y = b^{xy}$

⑥ $\dfrac{b^x}{b^y} = b^{x-y}$ ⑧ $(ab)^x = a^x b^x$