Math 242

Final Exam

Exams 1 to 4

5 probs each (Variations)

→ 20 probs total @ 10 pts each

→ 190 pts = 100%

Key

\# Is on the final

~~#~~ Not on final

? Maybe on final?

MATH 242 ... EXAM 1

1) A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a 5 week-period. State the domain, codomain, and range of your function. Where is it continuous? Where is it discontinuous?

2) Determine whether f is even, odd, or neither ...

a) $f(x) = x/(x^2+1)$

b) $f(x) = x/(x+1)$

3) Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, or trigonometric function and state their natural domain.

a) $f(s) = s^{1/4}$ (+) e^x, $\ln x$

b) $g(t) = t^3/(t^2-1)$

c) $h(x) = \tan(x) - \cos(x)$

d) $f(x) = x^3(1-x^2)$

4) On one set of xy-axis, graph and label each of the given functions ...

$f(x) = x^{1/4}$, $g(x) = \cos(x)$, and $h(x) = x^3$

5) Graph the curve $f(x) = 4\cos(\pi(x-2)) - 3$

6) For $f(x) = 3x+5$ and $g(x) = (x+1)/(x-1)$ find the given function and state its domain ...

a) $(f-g)(x)$

b) $(g \circ f)(x)$

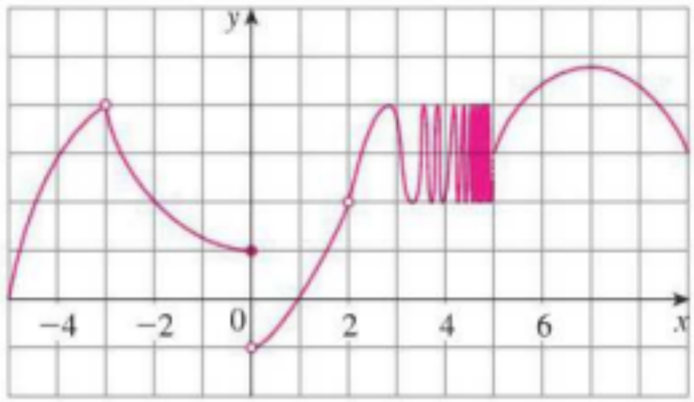
7) A ship is moving at a speed of 30 km/h parallel to a straight shoreline. The ship is 6 km from shore and its passes a lighthouse at noon.

a) Express the distance $s = f(d)$ between the lighthouse and the ship as a function of d, the distance the ship has traveled since noon.

b) Express $d = g(t)$ as a function of t, the time elapsed since noon.

c) Find $(f \circ g)(t)$. What does this function represent?

Use the following graph for problems 8 and 9.



8) For the function f whose graph was given above, state the value of each quantity, if it exists.

a) $\lim_{x\to -3^-} f(x)$ b) $\lim_{x\to -3^+} f(x)$ c) $\lim_{x\to -3} f(x)$ d) $f(-3)$

e) $\lim_{x\to 0^-} f(x)$ f) $\lim_{x\to 0^+} f(x)$ g) $\lim_{x\to 0} f(x)$ h) f(0)

9) For the function f whose graph was given above, state the value of each quantity, if it exists.

a) $\lim_{x\to -5^-} f(x)$ b) $\lim_{x\to -5^+} f(x)$ c) $\lim_{x\to -5} f(x)$ d) $f(5)$

10) Find the limit by using a rough sketch of the function ...

$$\lim_{x\to 3} \frac{2}{|x-3|}$$

11) Prove that the given limit does not exist ...

$$\lim_{x\to 0} \frac{|x|}{x}$$

12) Evaluate the limit and justify each equality you use by indicating the appropriate Limit Law(s) ...

$$\lim_{x\to 1} (x^2-3x)(x^3-2x+3)$$

13) Evaluate the limit, if it exists ...

$$\lim_{x\to 5} \frac{x^2-6x+5}{x-5}$$

14) Evaluate the limit, if it exists ...

$$\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$$

15) State the Squeeze Theorem and illustrate it with a figure.

16) Prove the statement using the ε, δ definition of a limit.

one linear function ε, δ proof

$$\lim_{x \to 1} 3x + 5 = 8$$

17) Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 1} 1 - 3x = -2$$

18) Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 2} x^2 = 4$$

19) Locate the discontinuities of the functions ...

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$

a) $f(x) = \tan(x)/\sqrt{4 - x^2}$

b) $f(x) = \cos(\sin(\cos(x)))$

cont?

discont?

20) Use continuity to evaluate the limit ...

$$\lim_{x \to \pi} \sin(x + \sin(x))$$

21) Does $x^5 - 10x^2 + 5 = 0$ have a root between $x = 0$ and $x = 2$? For either yes or no, you must explain why.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

MATH 242 ... EXAM 2

1) Use the limit definition to find the derivative of $f(x) = 2x - 3$

have one $f'(x)$ by limit definition.

2) Use the limit definition to find the derivative of $f(x) = \sqrt{1 - x}$

3) Use the limit definition to prove the Sum Rule: $\frac{d}{dx}(f + g)(x) = f'(x) + g'(x)$

4) Differentiate and do not simplify: $f(u) = 3u^{1/2} + u^2 + 3\cos(u)$

know all rules (and e^x, $\ln(x)$) to this section

5) Differentiate and do not simplify: $f(x) = \sqrt{x} \cdot \sin(x) - \tan(x)$

6) Differentiate and do not simplify: $f(y) = (y^{2/3} + 4y)/(2y^3 - y + 1)$

7) Differentiate and do not simplify: $f(x) = (x+2)/(2x - \tan(x))$

8) Differentiate and do not simplify: $f(t) = t^2 \cos(t) \sin(t)$

9) Differentiate and do not simplify: $f(x) = \sqrt{3x^2 - \sin(x^3)}$

10) Differentiate and do not simplify: $f(u) = \sin(\tan(2u+1))$

11) Differentiate and do not simplify: $f(t) = \tan^2(t + \sin(2t-1))$

12) Differentiate and do not simplify: $f(x) = \left(\frac{x^2+3x}{x^2+4}\right)^4$

13) Differentiate and do not simplify: $f(x) = \frac{\sin(x^2)+2x}{x - x^{4/3}}$

14) Find an equation of the tangent line to $x^2 - xy = y^2 + 1$ at the point (2,1)

15) Find y'' by implicit differentiation for $\sin(y) + \cos(x) = 1$

16) A company found the cost (in dollars) of producing x items is $C(x) = 10,000 + 5x + 0.01x^2$. What is the marginal cost when they produce 100 items?

17) Invasive species often display a wave of advance as they colonize new areas. Mathematical models based on random dispersal and reproduction have demonstrated that the speed with which such waves move is modeled by the function $v(r) = 2\sqrt{Dr}$, where r is the reproductive rate of individuals and D is a parameter quantifying dispersal. Calculate the derivative of the wave speed with respect to the reproductive rate r and explain its meaning.

18) A water tank has the shape of an inverted circular cone with base radius of 2m and a height of 4m. If water is being pumped into the tank at a rate of $1m^3/\min$, find the rate at which the water level is rising when the water is 2m deep.

Variation

19) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

20) Find the quadratic approximation to $f(x)=x^{1/3}$ near $x=8$

$\Delta x = dx$

2
6

21) If a cost function for producing an item x inches long is found to be

$$C(x)=100+5x-0.1x^2+0.01x^3$$

9.9 10 10.1

$\boxed{\Delta C \approx dC}$

what is the relative error in Cost for an item that is 10 in ± 0.1 in

$\dfrac{\Delta x}{x} = \dfrac{\text{error}}{\text{value}}$ $\quad \dfrac{\Delta C}{C} \quad \therefore \dfrac{dC}{C} = \dfrac{C'(x)\,dx}{C(x)}$

Math 242 ... Exam 3

1) Find the absolute maximum and minimum values of the function on the given interval

$$f(x)=3x^4-4x^3-12x^2+1, \qquad -1\leqslant x\leqslant 1$$

one abs. extrema prob.

2) Find the absolute maximum and minimum values of the function on the given interval

$$f(x)=x^3-3x^2+1, \qquad -1\leqslant x\leqslant 3$$

2
0

3) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x)=x^3-x, \qquad 0\leqslant x\leqslant 2$$

4) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x)=x^{1/3}, \qquad 0\leqslant x\leqslant 1$$

5) For $f(x)=2+3x^2-x^3$ find the domain, the y-intercept, and any asymptotes.

one function problem.

6) For $f(x)=2+3x^2-x^3$ find any critical numbers, intervals of increase or decrease, and any local extrema.

7) For $f(x)=2+3x^2-x^3$ find any inflection points, intervals of concave up or down, and plot the function.

8) For $f(x) = x/(x-1)$ find the domain, the intercepts, and any asymptotes.

9) For $f(x) = x/(x-1)$ find any critical numbers, intervals of increase or decrease, and any local extrema.

10) For $f(x) = x/(x-1)$ find any inflection points, intervals of concave up or down, and plot the function.

11) For $f(x) = (x-3)\sqrt{x}$ find the domain, the intercepts, and any asymptotes.

12) For $f(x) = (x-3)\sqrt{x}$ find any critical numbers, intervals of increase or decrease, and any local extrema.

13) For $f(x) = (x-3)\sqrt{x}$ find any inflection points, intervals of concave up or down, and plot the function.

? 14) A rectangular storage container with an open top is to have a volume of $8m^3$. The length of its base is twice the width. Material for the base costs 2 dollars per square meter. Material for the sides cost 1 dollar per square meter. Find the width of the cheapest such container.

? 15) The rate at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100I}{I^2 + I + 4}$$

where I is the light intensity. For what light intensity is P a maximum?

16) Create a formula to find the $\sqrt{c}$ using Newton's Method. What number should you stay away from for your first guess?

?

17) Create a formula to find the root of $f(x) = \frac{1}{x} - a$ using Newton's Method.

18) Find the most general antiderivative of the function

$$f(x) = 2\sin(x) - x^2 + x^{1/2}$$

19) Find the most general antiderivative of the function

$$f(t) = 1 - \sec(t)\tan(t) + \cos(t)$$

20) Find the most general antiderivative of the function

$$f(s) = 3s^{3/4} + \sec^2(s)$$

21) Find $f(x)$ if

$$f''(x) = -1 + 12x - 12x^2, \qquad f'(0) = 12, \qquad f(0) = 4$$

MATH 242 ... EXAM 4

Area $\approx \left(f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*) \right) \Delta x$

1) Use the definition

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and right endpoints $x_i = a + i\Delta x$ to evaluate the integral

$$\int_0^1 (x^2 + x)dx$$

2) Find an approximation to the integral $\int_1^5 (x^2 + x)dx$ using a Riemann sum with right endpoints and $n = 4$.

3) Evaluate the indefinite integral

$$\int (2\sin(x) - x^2 + x^{1/2})\,dx$$

4) Evaluate the indefinite integral

$$\int (1 - \sec(t)\tan(t) + \cos(t) - \csc^2(t))\,dt$$

5) Evaluate the indefinite integral

$$\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\,dx$$

? 6) Evaluate the indefinite integral

$$\int s\sqrt{s+2}\,ds$$

? 7) Evaluate the indefinite integral

$$\int \frac{\cos(\pi/x)}{x^2}\,dx$$

? 8) Evaluate the definite integral

$$\int_0^1 (x^4 - 8x + 1 - \sqrt{x})\,dx$$

? 9) Evaluate the definite integral

$$\int_0^{\pi/2} (2x + 1 + \cos(x))\,dx$$

? 10) Evaluate the definite integral

$$\int_0^2 t^2 \sin(t^3)\,dt$$

? 11) Evaluate the definite integral

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$$

(plus: e^x, $\ln x$ type probs)

? 12) Evaluate the definite integral

$$\int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx$$

? 13) Setup, but do not evaluate, the definite integrals to find the area of the region bounded by the curves $y = \sin(x)$, $y = \cos(x)$, $x = 0$, and $x = \pi$. Include a sketch of the region.

Area

14) Find the area enclosed by the curves $y = x^2$ and $y = 4x - x^2$.

? 15) Setup an integral to find the volume of a pyramid whose base is a square with side L and whose height is h.

Volume

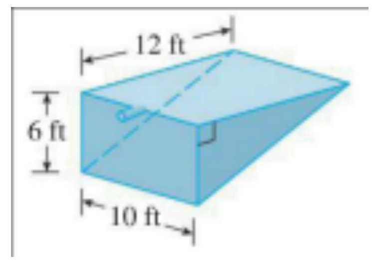
16) Setup an integral to find the volume of the solid obtained by rotating the region bounded by $y=x^4$, $y=0$, and $x=2$ about $x=-1$ using the method of disks.

17) Find the volume of the solid obtained by rotating the region bounded by $y=x^4$, $y=0$, and $x=2$ about $x=-1$ using the method of shells.

18) A force of 10N is required to hold a spring that has been stretched from its natural length of 10cm to a length of 20cm. How much work is done in stretching the spring from 20cm to 40cm?

Work

19) Setup an integral to find the work required to pump the water out of the given tank. Let ρ be the density of water in lb/ft^3.



20) Find the average value of $f(x)=x+\sqrt{x}$ over the interval $[0,4]$

f_{ave}

21) Find c such that $f_{\text{ave}}=f(c)$ for $f(x)=x\sqrt{1-x^2}$ over the interval $[0,1]$.

we will have one f_{ave}