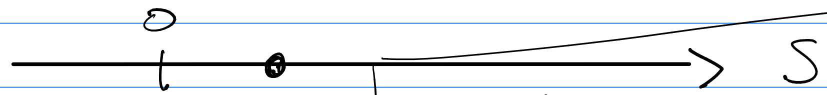
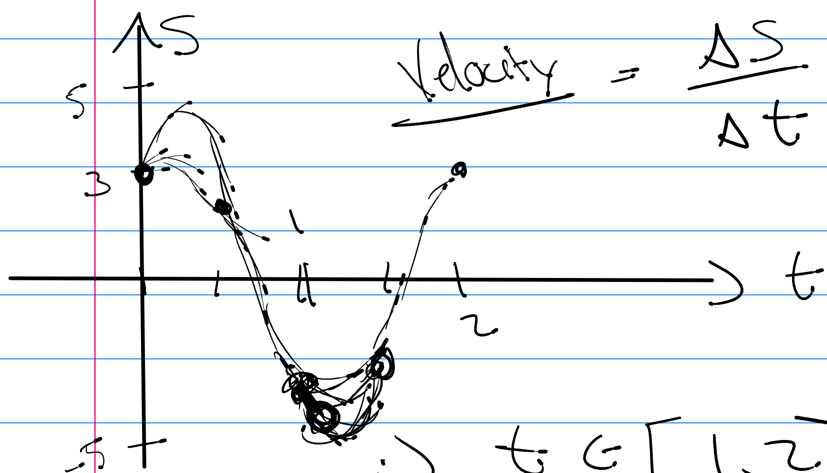
# Math 242

Q's



$$\boxed{S(t) = 2\sin(\pi t) + 3\cos(\pi t)}$$



Velocity $= \dfrac{\Delta S}{\Delta t}$

| t | S |
|---|---|
| 0 | 3 |
| 1/2 | 2 |
| 1 | -3 |
| 3/2 | |
| 2 | |

i) $t \in [1,2]$ $\quad V = \dfrac{S(2)-S(1)}{2-1}$

ii) $t \in [1, 1.1]$ $\quad V = \dfrac{S(1.1)-S(1)}{.1} = 10(S(1.1)-S(1))$

iv) $[1, 1.001]$ $\quad V = \dfrac{S(1.001)-S(1)}{.001} = 1000(\square)$

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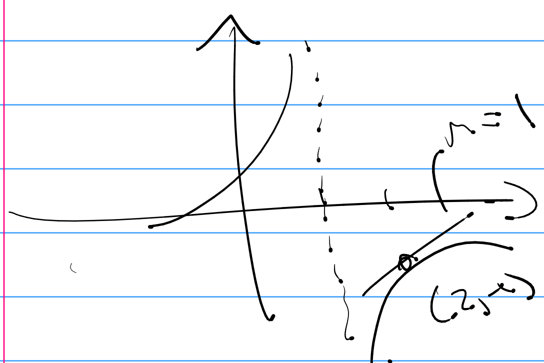
1.3 (3c) $\quad f(x) = \dfrac{1}{1-x}$ $\quad P(2,-1)$ is on it

b) slope of tangent @ $P(2,-1)$ was $m = 1$



$$\boxed{y - y_0 = m(x - x_0)}$$

$$y + 1 = 1(x-2)$$

$$\boxed{y = x - 3}$$

1.5 #27 $\lim_{x \to 0} \frac{\cos 2x - \cos x}{x^2}$

see video

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1.5 #40 $y = \frac{x^2+1}{3x-2x^2} = \frac{x^2+1}{x(3-2x)}$

holes? root of num (+) root of denom.

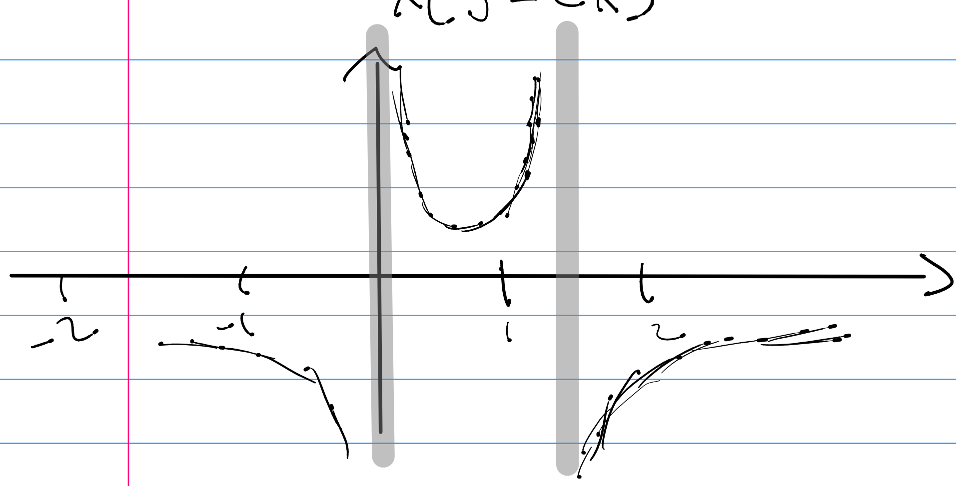
ex $\frac{x+1}{x^2-1} = \frac{\cancel{(x+1)}}{(x+1)(x-1)} = \frac{1}{x-1}$, $\underline{x \neq -1}$

vert. asym root of denom (+) $\boxed{\text{not}}$ of num.

ex $x=1$ vert. asym.

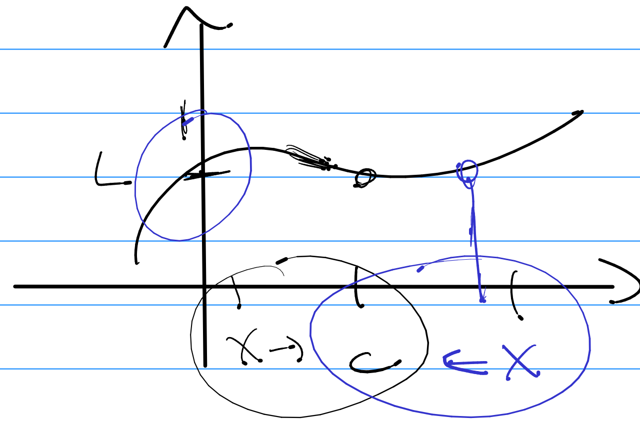
$\lim_{x \to 1^-} f(x) = \pm\infty$, $\lim_{x \to 1^+} f(x) = \pm\infty$

#40 $f(x) = \frac{x^2+1}{x(3-2x)}$ asym. $x=0$, $x = 3/2$



$\boxed{\lim_{x\to c^-} f(x)}$ vs $\boxed{\lim_{x\to c^+} f(x)}$ vs $\boxed{\lim_{x\to c} f(x)}$

both

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$x = 2/3$

$\boxed{2/3}$

$\boxed{3x-2=0}$

$3x=2$

$x=2/3$

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$x^2-4=0$

$(x-2)(x+2)=0$

$x-2=0 \quad x+2=0$

$x=2 \quad x=-2$

$a\cdot b=0$

$a=0 \quad b=0$

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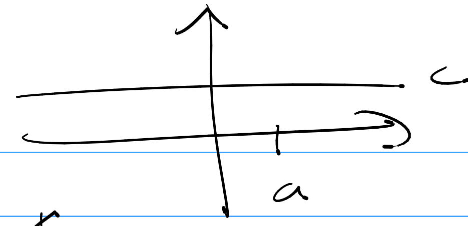
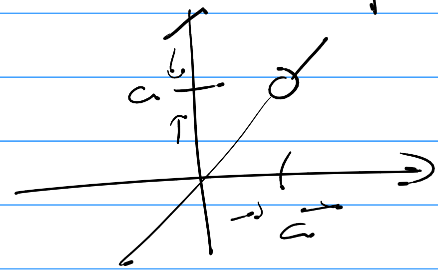
Same idea for limits.

$$\lim_{x\to 3} \frac{(x)^{1/2}+2x^3-x}{4x-2/x}$$

$$\lim_{x \to a} c = c$$




$$\lim_{x \to a} x = a$$

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(1) $$\lim_{x \to a} f(x) \pm g(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$\lim_{x \to 3} (x) + (\pi) = \boxed{3 + \pi}$$

(2) $$\lim_{x \to a} f(x) \cdot g(x) = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$$

ex $$\lim_{x \to -10} (3x) + (e) = 3(-10) + e = \boxed{e - 30}$$

(3) $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \left(\lim_{x \to a} g(x) \neq 0\right)$$

ex $$\lim_{x \to 3} \frac{2x + 4}{x - 1} = \frac{2(3) + 4}{3 - 1} = \boxed{5}$$

(4) $$\lim_{x \to a} (f(x))^p = \left(\lim_{x \to a} f(x)\right)^p$$

$p = 1, 2, 3, \ldots$

$p = 1/2, 1/3, 1/4, \ldots$

$$\lim_{x \to 2} \frac{\left((x)^{1/2} + 3x^2 - x\right)}{\left(3 - 4/x\right)} = \frac{(2)^{1/2} + 3(2)^2 - (2)}{3 - 4/2}$$

$$= \frac{10 + \sqrt{2}}{1} = \boxed{10 + \sqrt{2}}$$

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ex $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

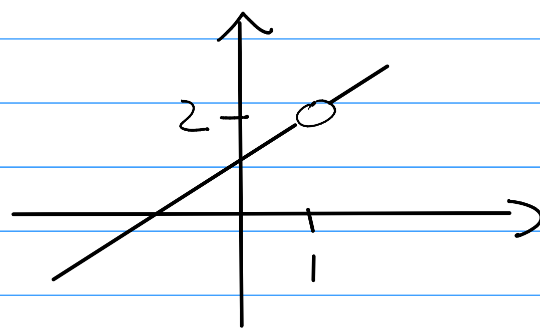
$$f(x) = \frac{x^2 - 1}{x - 1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$f(x) = x + 1, \quad x \neq 1$$

$$= \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} x + 1 \quad \left(, \underline{x \neq 1}\right)$$

$$= 1 + 1 = \boxed{2}$$

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<u>Substitution Rule</u>, $p(x)$ is a polynomial

$$\lim_{x \to a} p(x) = p(a)$$

ex $\lim_{x \to -1} 3x^2 - x + 2 = 3(-1)^2 - (-1) + 2$

$$= \boxed{6}$$

Ex $f(x) = 2x^2 - x + 1$

$f(\boxed{3}) = 2[\ 3\ ]^2 - [\ 3\ ] + 1$

$$\lim_{h\to 0} \frac{\boxed{f(3+h)} - \boxed{f(3)}}{h} = \lim_{h\to 0} \frac{[2(3+h)^2 - (3+h) + 1] - [16]}{h}$$

$$= \lim_{h\to 0} \frac{[2(9+6h+h^2) - 3 - h + 1] - 16}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{18} + \boxed{12h + 2h^2} - \cancel{2} - \boxed{h} - \cancel{16}}{h} = \lim_{h\to 0} \frac{\cancel{h}(11+2h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 11 + 2h \quad (h \neq 0) = \boxed{11}$$

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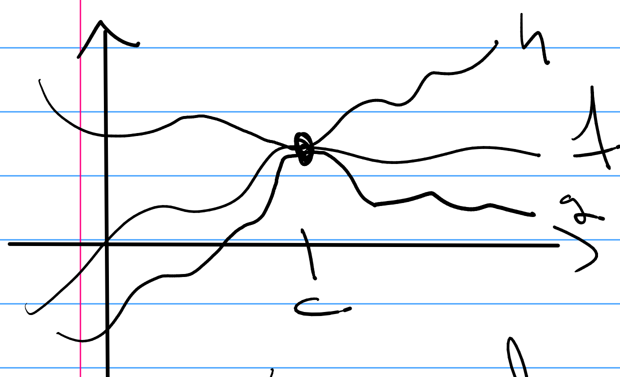



$g(x) \le f(x) \le h(x)$

$\lim_{x\to c} f(x) = ?$

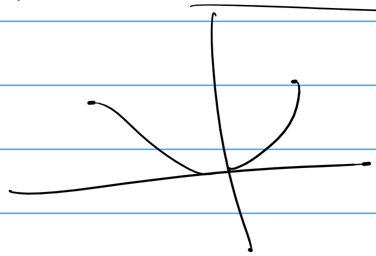
Know: $\lim_{x\to c} h(x) = L$, $\lim_{x\to c} g(x) = L$

Squeeze th$^m$ $\rightarrow \lim_{x\to c} f(x) = L$

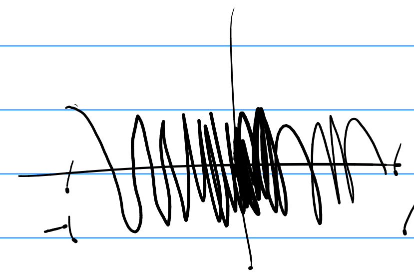
typed

$-1 \le \sin(x) \le 1$ (fact)

$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$



$\sin\left(\frac{1}{x}\right)$



$-1 \le \sin(\theta) \le 1$

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$ $\quad x \neq 0$

$$\left.\begin{aligned} \lim_{x \to 0} (-x^2) &= -0^2 = 0 \\ \lim_{x \to 0} x^2 &= 0^2 = 0 \end{aligned}\right] \Rightarrow \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$