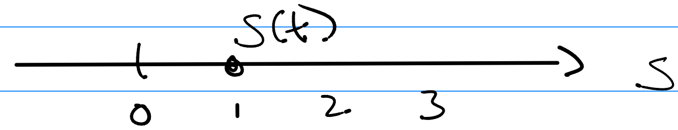
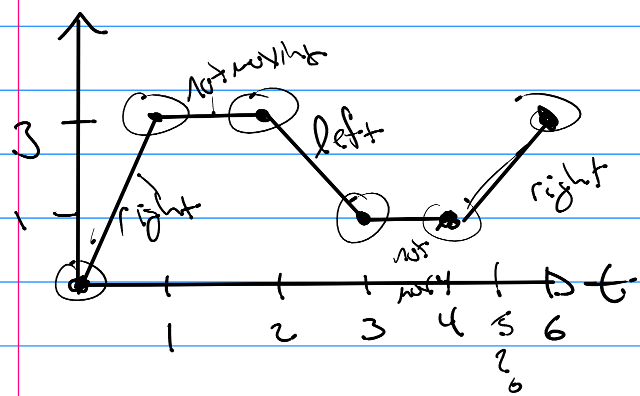
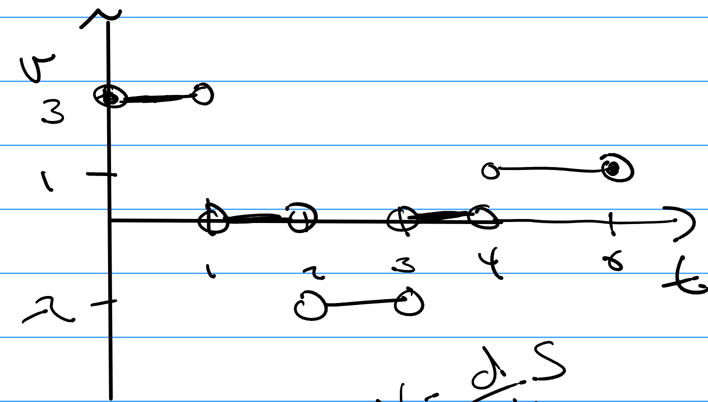
Math 242

Q's/ 2.1 #11

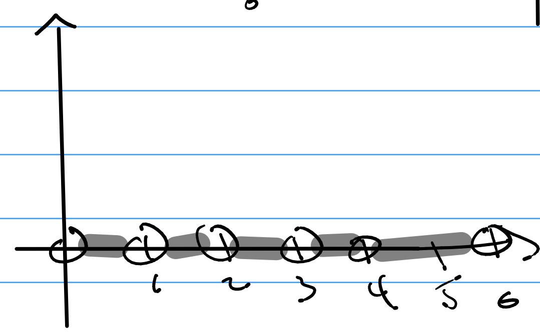






$V = \frac{d}{dt} S$

$\frac{d}{dt}(v) = a$



$$\boxed{D_x[f(x)] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$D_x[f(x)]\Big|_{x=a} = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

Use the real Definition of $\frac{d}{dx}[f(x)]$ to find $f'(x)$ on basic functions

→ Gives Derivative formulas.

$$D_x[3] = 0$$

ESC

$$D_x[x^{2/3}] = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-1/3}$$

$$D_x\left[\frac{2}{3}x^{-1/3}\right] = \frac{2}{3}\left[-\frac{1}{3}x^{-1/3-1}\right] = -\frac{2}{9}x^{-4/3}$$

$$[x^{2/3}]' = \frac{2}{3}x^{-1/2}$$

$$[x^{2/3}]'' = -\frac{2}{9}x^{-4/3} = \left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)x^{-4/3}$$

$$[x^{2/3}]^{(3)} = \left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)x^{-7/3}$$

$$[x^{2/3}]^{(4)} = \left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)x^{-10/3}$$

$$= \frac{(2)(-1)(-4)(-4)}{3^4}x^{-10/3}$$

$$[x^{2/3}]^{(n)} = ?$$

$$D_x[f+g] = f' + g'$$

$\boxed{\text{Know}}$

$$D_x[fg] = f'g + fg'$$

$$D_x[f/g] = \frac{f'g - fg'}{g^2}$$

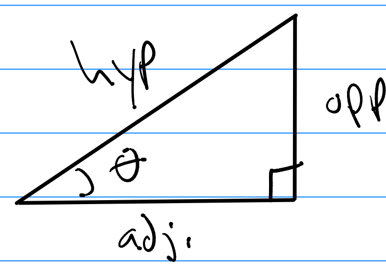
$$D_x\left[\frac{3x+2}{x^{1/2}-1}\right] = \frac{D_x[3x+2](x^{1/2}-1) - (3x+2)D_x[x^{1/2}-1]}{(x^{1/2}-1)^2}$$

$$= \frac{(D_x(3x) + D_x(2))(x^{1/2}-1) - (3x+2)(D_x(x^{1/2}) - D_x(1))}{(x^{1/2}-1)^2}$$

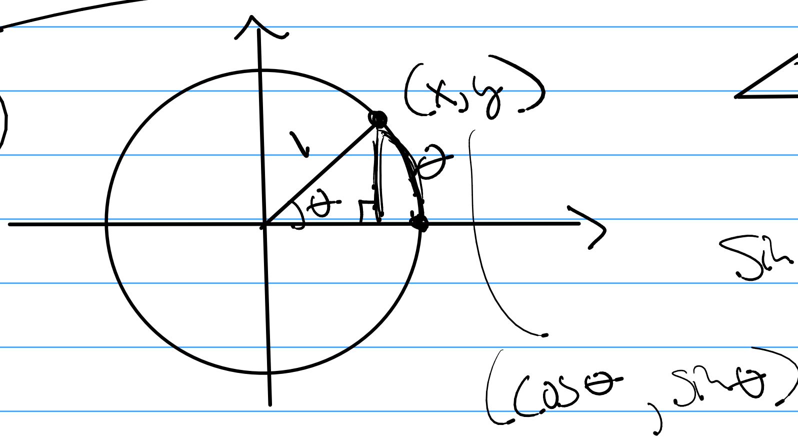
$$= \boxed{\frac{3(x^{1/2}-1) - (3x+2)(\frac{1}{2}x^{-1/2})}{(x^{1/2}-1)^2}}$$

Trig Functions

2.4

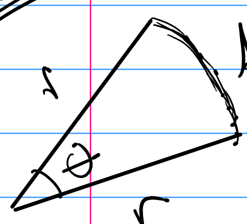


Note:



$\sin\theta = \frac{opp}{hyp}$, $\cos\theta = \frac{adj}{hyp}$

also



Arc $= \theta r$

$C = (2\pi) r$

$adj^2 + opp^2 = hyp^2$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad\csc\theta = \frac{1}{\sin\theta}$

$$D_x[\sin x] = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

<u>Know:</u> $\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

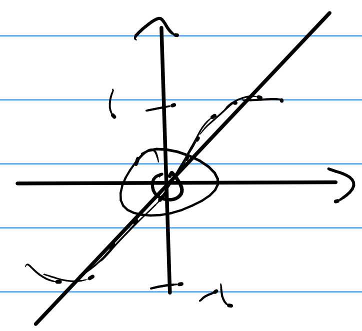
$$D_x[\sin x] = \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

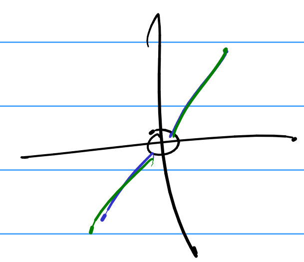
$$= \lim_{h\to 0} \sin x\left[\frac{\cos h - 1}{h}\right] + \cos x\left[\frac{\sin h}{h}\right]$$

$$= \sin x \lim_{h\to 0} \frac{\cos(h) - 1}{h} + \cos x \boxed{\lim_{h\to 0} \frac{\sin(h)}{h}}$$

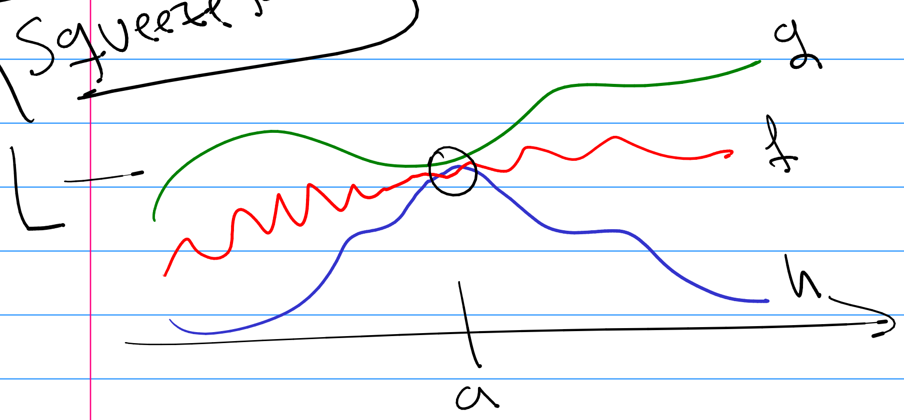
$\frac{0}{0} = 1$

$\sin(h)$ vs h





Squeeze th^m

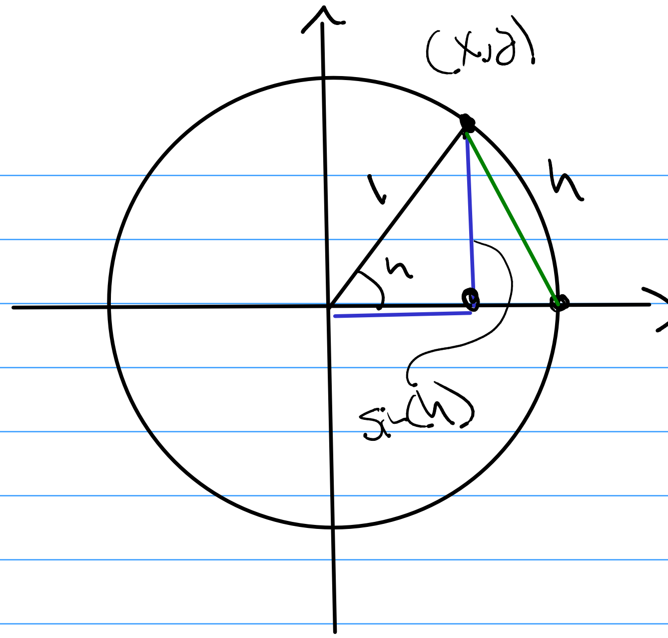


$\lim_{x\to a} g = \lim_{x\to a} h = L$

$\Rightarrow \lim_{x\to a} f = L$

<u>For</u> $\lim_{h\to 0} \frac{\sin h}{h} = \frac{0}{0} = 1$

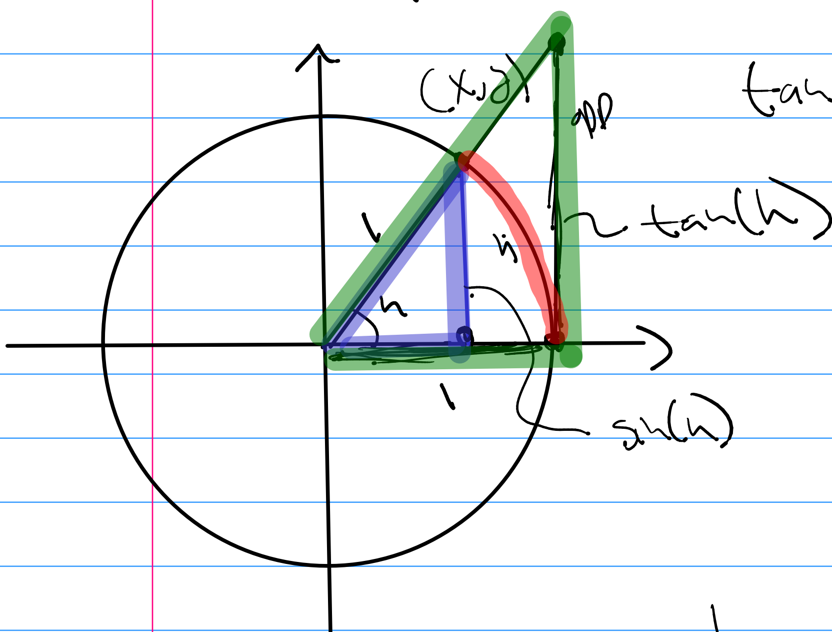
back to unit circle



blue < green < arc

$\sin(h) < h$

so $\dfrac{\sin(h)}{h} < 1$



$\tan(h) = \dfrac{opp}{adj} = opp$

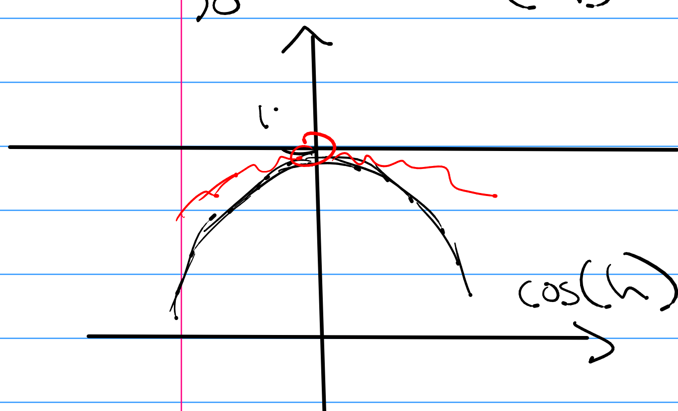
large side $> h$

$\tan h > h$

$\dfrac{\sin h}{\cos h} > h$

so $\dfrac{\sin h}{h} > \cos h \rightsquigarrow \cos h < \dfrac{\sin h}{h}$

so $\cos(h) < \dfrac{\sin h}{h} < 1$



$\lim_{h \to 0} \cos(h) = \cos(0) = 1$

$\lim_{h \to 0} 1 = 1$

$\therefore \lim_{h \to 0} \dfrac{\sin(h)}{h} = 1$

So

(see below)

$$D_x[\sin x] = \sin(x)\left(\lim_{h\to 0}\frac{\cos(h)-1}{h}\right)^{\to 0} + \cos(x)\lim_{h\to 0}\frac{\sin(h)}{h}^{\to 1}$$

ok

$$\lim_{h\to 0}\frac{(\cos(h)-1)}{h}\frac{(\cos(h)+1)}{(\cos(h)+1)} = \lim_{h\to 0}\frac{\cos^2(h)-1^2}{h(\cos(h)+1)}$$

b/c $\sin^2(h)+\cos^2(h)=1$

$$\Rightarrow \cos^2(h)-1 = -\sin^2(h)$$

$$\lim_{h\to 0}\frac{-\sin^2(h)}{h(\cos(h)+1)} = -\lim_{h\to 0}\left(\frac{\sin(h)}{h}^{\to 1}\cdot\frac{\sin(h)^{\to 0}}{\cos(h)+1_{\to 2}}\right)$$

$$= -\left(1\cdot\frac{0}{2}\right) = 0$$

$$\boxed{D_x[\sin(x)] = \cos(x)}$$

$$D_x[\cos(x)] = \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h} = \text{similar as above}$$

$$\boxed{D_x[\cos(x)] = -\sin(x)}$$

We can use all our formulas now to also show..

$$D_x[\tan(x)] = \sec^2(x)$$

$$D_x[\cot(x)] = -\csc^2(x)$$

$$D_x[\sec(x)] = \sec(x)\tan(x)$$

$$D_x[\csc(x)] = -\csc(x)\cot(x)$$

Note: $\lim\limits_{\square \to 0} \frac{\sin(\square)}{\square} = 1$

use this: ex $\lim\limits_{x \to 0} \frac{\sin(6x)}{6x} = 1$

same as $6x \to 0$

ex $\lim\limits_{x \to 0} \pi \left(\frac{\sin(\pi x)}{\cancel{\pi x}} \right) = \pi \cdot 1 = \boxed{\pi}$

$$\lim_{x \to 0} \frac{\tan(3x)}{x} = \lim_{x \to 0} 3 \cdot \left(\frac{\sin(3x)}{\cancel{3x}} \right) \frac{1}{\cos(3x)}$$

$$= 3 \cdot 1 \cdot \frac{1}{1} = \boxed{3}$$