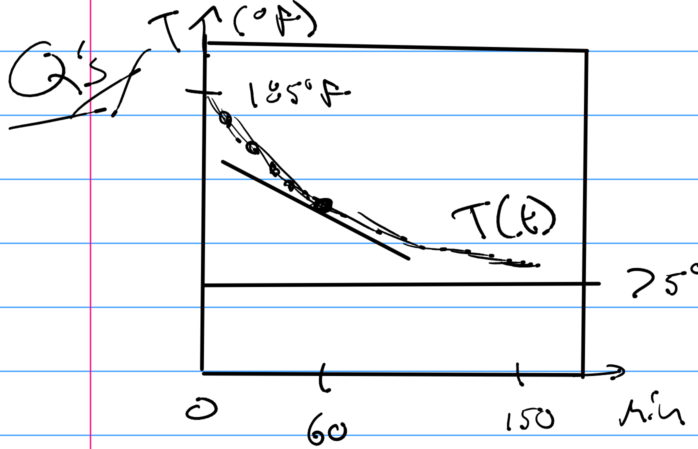
Math 242

Q's



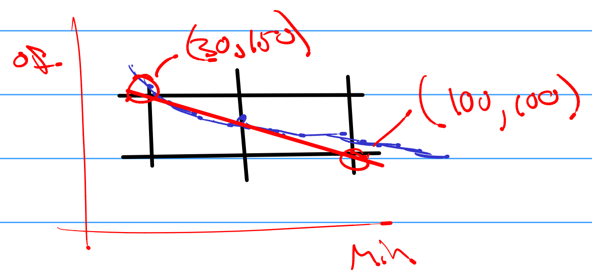
Estimate rate of change of the Temp after 1 hr.

inst. rate of change @ 60 min

→ find $\boxed{T'(60)}$

Estimate $T'(60)$ ... How?

try #1 eyeball a tangent line, estimate its slope



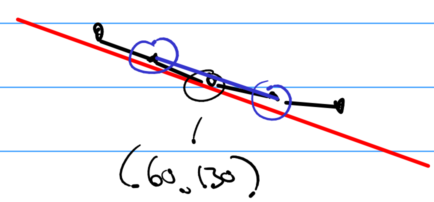
$\sim \frac{50}{-70} = -\frac{5}{7} \frac{°F}{\text{Min}}$

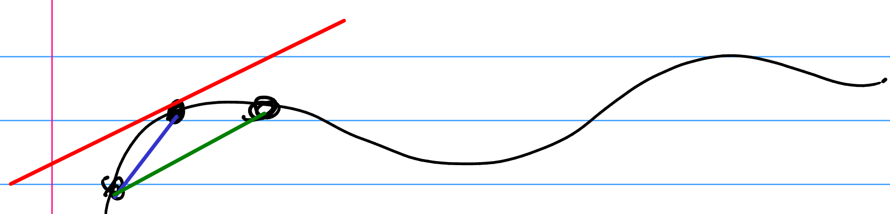
try #2 and you had the data (table of values)

ex

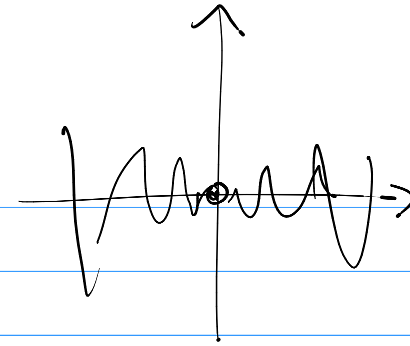
t	T
40	143°F
50	136°F
60	130°F
70	124°F
80	119°F
⋮	

$\frac{dT}{dt}@60 \sim \frac{\Delta T}{\Delta t}$





$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$



Note: when does $f'(a)$ exist?

$$f'(a) = \boxed{\lim_{x\to a} \frac{f(x)-f(a)}{x-a}} \text{ exists}$$

$\rightarrow$ continuous function. $f(a) = \lim_{x\to a} f(x)$

is $f(x)$ cont? $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

$$\boxed{-1 \le \sin\left(\frac{1}{x}\right) \le 1}$$

$\lim_{x\to 0^+} x\sin\left(\frac{1}{x}\right)$

x is pos

$-x \le x\sin\left(\frac{1}{x}\right) \le x$

$\lim_{x\to 0^+} -x = 0$

$\lim_{x\to 0^+} x = 0$

$\lim_{x\to 0^+} x\sin\left(\frac{1}{x}\right) = 0$

$\lim_{x\to 0^-} x\sin\left(\frac{1}{x}\right)$

b/c x is neg

$-x \ge x\sin\left(\frac{1}{x}\right) \ge x$

$\lim_{x\to 0^-} -x = 0$, $\lim_{x\to 0^-} x = 0$

$\lim_{x\to 0^-} x\sin\left(\frac{1}{x}\right) = 0$

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\cancel{x}\sin(\frac{1}{x}) - 0}{\cancel{x}}$$

$$\lim_{x\to 0} \sin(\tfrac{1}{x}) \text{ dne} \quad \text{So } \boxed{f'(0) \text{ dne}}$$

$\boxed{2.7}$ 18 tank. Volume of 5000 gallons

$$V(t) = 5000\left(1 - \frac{1}{40}t\right)^2 \qquad t \in [0, 40]$$

rate of drain after 5 mins? $V'(5)$

1 $V'(t) = D_t\left[5000\boxed{\left(1-\frac{1}{40}t\right)^2}\right]$

$\frac{\Delta V}{\Delta t}$ $= 10000\left(1-\frac{1}{40}t\right)^1 D_t\left[1-\frac{1}{40}t\right]$

$= -250\left(1-\frac{1}{40}t\right) \frac{\text{gal}}{\text{min}}$

$V'(t) = (6.25t - 250)$ gal/min $\quad t \in [0, 40]$

2 $V'(5) =$

$V'(10) =$?

$V'(20) =$

$V'(40) =$

$\boxed{\text{Summary}}$?

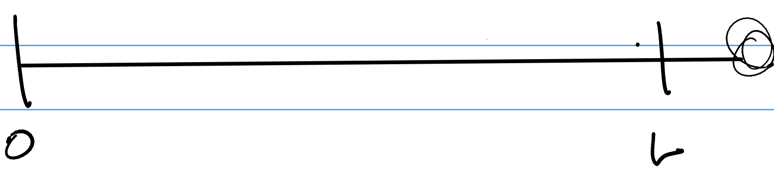
#28 $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}}$

L: length of string
T: tension
ρ: linear density.

rate of change of f
with respect to L (if T, ρ are fixed)

$$f'(L) = \frac{d}{dL}\left[\frac{1}{2L}\sqrt{\frac{T}{\rho}}\right] = \frac{1}{2}\sqrt{\frac{T}{\rho}}\,\frac{d}{dL}\left[\frac{1}{L}\right]$$

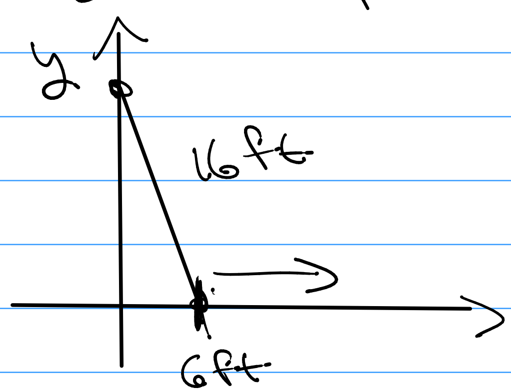
$$f'(L) = \frac{1}{2}\sqrt{\frac{T}{\rho}}\,\frac{d}{dL}\left[L^{-1}\right] = \boxed{-\frac{1}{2}\sqrt{\frac{T}{\rho}}\,\frac{1}{L^2}}$$



$$\frac{df}{dL} \propto \ominus \frac{1}{L^2}$$

Related Rates 2.8

① Given implicit functions in an equation.



$\boxed{x^2 + y^2 = 16^2}$ position eqn

$\to x = f(t),\ y = g(t)$

Knew $\dfrac{dx}{dt} = 3\text{ ft/sec}$

② Implicit Derivs. to find $\boxed{\text{related rates eqn}}$

with respect to ind. variable

(ex) $x^2+y^2=16^2$ but $x=f(t)$, $y=g(t)$

x, y are fn ft

t is in seconds ← ind. variable

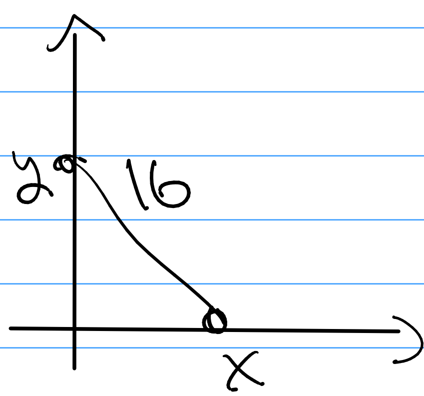
$$D_t[x^2+y^2]=D_t[16^2]$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\boxed{x\frac{dx}{dt}+y\frac{dy}{dt}=0}$$ related rates equation

(3) use position and related rates eqn's to solve problems.

(ex)



$x(0)=6$ ft

$\frac{dx}{dt}=3$ ft/sec

$x^2+y^2=16^2$

$x\frac{dx}{dt}+y\frac{dy}{dt}=0$

how fast is y fall

Q: $\frac{dy}{dt}=?$

So

$6^2+y^2=16^2 \rightarrow y=\sqrt{16^2-6^2}$

$6(3)+y\left(\frac{dy}{dt}\right)=0$

$$\frac{dy}{dt}=\frac{-18}{\sqrt{16^2-6^2}}\ \text{ft/sec}$$