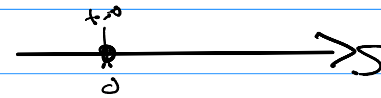
Math 242

Q's / 2.7 #2 $f(t) = 0.01t^4 - 0.04t^3$



a) $v(t) = 0.04t^3 - 0.12t^2$ $\quad a(t) = 0.12t^2 - 0.24t$

b) $v(1) = 0.04(1)^3 - 0.12(1)^2 = -0.08$ u/s

c) rest? $v(t) = 0$ so $0.04t^3 - 0.12t^2 = 0$

$4t^3 - 12t^2 = 0$

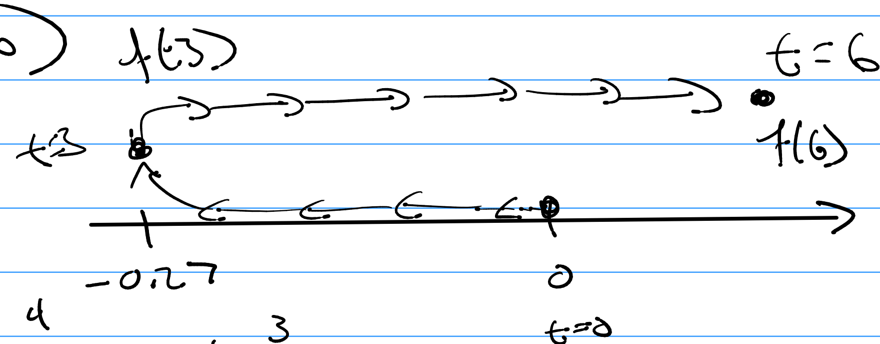
$4(t^2)(t-3) = 0$

$\underbrace{t = 0 \text{ sec.} \quad t = 3 \text{ sec.}}_{\text{rests}}$

also:
- rest @ 0 sec
- move left for 3 sec
- move right after

d) $t \in (3, +\infty)$ $f(3)$

e) total dist:



$f(3) = 0.01(3)^4 - 0.04(3)^3$

$= 0.81 - 1.08 = -0.27$

$f(6) = 0.01(6)^4 - 0.04(6)^3 =$

total dist: $(.27) + |f(6) - f(3)|$

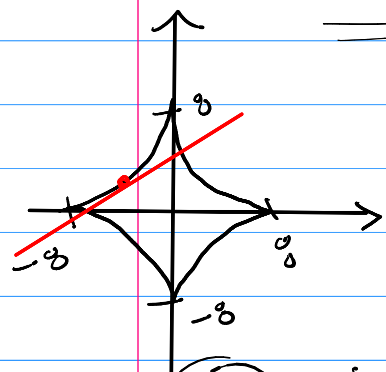
total dist: $|f(0) - f(3)| + |f(6) - f(3)|$

(i) Speeding up (positive accel) $a(t) = 0.12t^2 - 0.24t$

Slowing down (neg. accel)

Know this

2.6 #30 $x^{2/3} + y^{2/3} = 4$ $\left.\frac{dy}{dx}\right|_{(-3\sqrt{3},\,1)}$ $3\sqrt{3} = 3^{3/2}$



$y = +(4 - x^{2/3})^{3/2}$ use for explicit deriv.

a) implicit Derivatives $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$$x^{-1/3} + y^{-1/3}\frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}}$$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$ slope point $(-3\sqrt{3}, 1)$

$= (-3^{3/2}, 1)$

slope @ $(-3^{3/2}, 1)$

$$\frac{dy}{dx} = -\frac{1^{1/3}}{(-3^{3/2})^{1/3}} = \frac{1}{3^{1/2}} = \frac{1}{\sqrt{3}}$$

So eqn of tangent $\boxed{y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3})}$

$\boxed{2.6\ \#35}$ $\boxed{x^2 + 4y^2 = 4}$ $y'' = ?$

$$2x + 8yy' = 0$$

$$y' = \frac{-x}{4y}$$

So $y'' = D_x\left[\frac{-x}{4y}\right] = \frac{(-1)(4y) - (-x)(4)y'}{16y^2}$

$$y'' = \frac{-4y + 4x\left(\frac{-x}{4y}\right)}{16y^2} = \frac{-4y - \frac{x^2}{y}}{16y^2}$$

$$y'' = \frac{-4y^2 - x^2}{16y^3} = \boxed{-\frac{4y^2 + x^2}{16y^3}}$$ b/c 1st eqn

$$y'' = \frac{-4}{16y^3} = \boxed{-\frac{1}{4y^3}}$$ $x^2 + 4y^2 = 4$

Exam 2

21 probs @ 10 pts

200 pts = 100%

Ch 2 $\boxed{2.1/2.2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(3 probs) 1-3

$\boxed{f'(x) \text{ by limit}}$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$a^2 - b^2 = (a+b)(a-b)$

(ex) $f(x) = \sqrt{x}$ $\quad f'(x) = ?$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$= \underline{\text{etc}}$

(ex) $\frac{d}{dx}[f(x) \cdot g(x)]$ $\qquad$ one of these

$\frac{d}{dx}[f(x) + g(x)]$

$\boxed{2.3 \,/\, 2.4 \,/\, 2.5}$ Rules of Differentiation (10 probs)

(1) write the rules for

$\frac{d}{dx}[c]$, $\frac{d}{dx}[x]$, $\frac{d}{dx}[x^n]$, $\frac{d}{dx}[f(x)+g(x)]$

$\frac{d}{dx}[f(x)g(x)]$, $\frac{d}{dx}[f/g]$, $\frac{d}{dx}[f(g(x))]$

$\frac{d}{dx}[\sin(x)]$ $\;\cos x$, $\tan x$

(2-10) take derivative (do not simplify)

$$D_x[3x^2 + x^{1/2} + \sin x - \tan x]$$
$$= 6x + \tfrac{1}{2}x^{-1/2} + \cos x - \sec^2 x$$

(ex) $D_x\left[\sqrt{2x - \sin(x^3)}\right]$

(ex) $D_x\left[\dfrac{\sin(x^2) + 2x}{x - x^{4/3}}\right] =$

$$= \frac{\frac{d}{dx}\left[\sin(x^2) + 2x\right]\left(x - x^{4/3}\right) - \left(\sin(x^2) + 2x\right)\frac{d}{dx}\left[x - x^{4/3}\right]}{\left(x - x^{4/3}\right)^2}$$

$$= \boxed{\frac{\left(2x\cos(x^2) + 2\right)\left(x - x^{4/3}\right) - \left(\sin(x^2) + 2x\right)\left(1 - \frac{4}{3}x^{1/3}\right)}{\left(x - x^{4/3}\right)^2}}$$

2.6 Implicit Derivatives (2 probs)

1. eqn of tangent line type (see above)
2. y'' (see above)

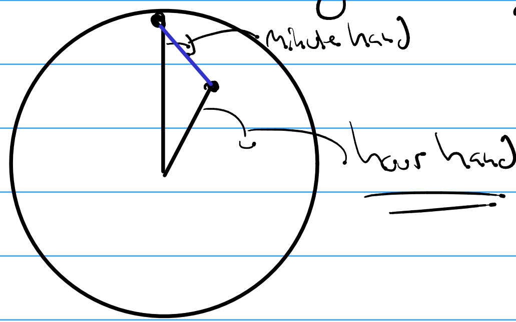
2.7 Apps in Sciences (2 word problems)

word probs: Economics, particle motion, like #18, #34

2.8 Related Rates (2 word problems)

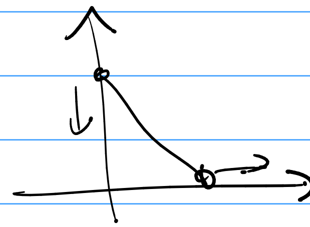
① Cone increasing in height (like #2a)

②



#50

③ falling ladder



2.9 Polynomial Approx and Differentials

① Quadratic Approx. of a function

ex $f(x) = \sqrt{x} \to f'(x) = \frac{1}{2\sqrt{x}} \quad f''(x) = \frac{-1}{4x^{3/2}}$

quad. approx of $f(x)$ near $x = 9$

$$f(x) \approx Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

$$\boxed{\sqrt{x} \approx 3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2}$$

② error and differentials

word problem given function, measure, error of measure

→ rel. error?

ex $C(x) = 2x^3 - \sqrt{x} + 10$

$x = 4 \pm 0.1$ $\quad x=4 \quad dx = \Delta x = 0.1$

rel. error $\quad \frac{\Delta C}{C} \approx \frac{dC}{C} = \frac{C'(x) \cdot dx}{C(x)}$
