Math 242

~~Qs~~ 4.3 #67

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right)}$$

$a=0$, $b=1$

$\Delta x = \frac{1}{n}$

Know: ☆ $\int_a^b f(x)\,dx = \boxed{\lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x \qquad \Delta x = \frac{b-a}{n}}$

$$\lim_{n\to+\infty} \sum_{i=1}^{n} \underbrace{\left(\frac{i^4}{n^4} + \frac{i}{n} \right)}_{f(x_i^*)} \underbrace{\left(\frac{1}{n}\right)}_{\Delta x} = \lim_{n\to+\infty} \sum_{i=1}^{n} \underbrace{\left(\left(\frac{i}{n}\right)^4 + \left(\frac{i}{n}\right) \right)}_{f(x) = x^4 + x} \underbrace{\left(\frac{1}{n}\right)}_{\Delta x}$$

right endpts! $x_i^* = \frac{i}{n}$

So $\int_0^1 (x^4+x)\,dx =$ #67's sum

$$\int_0^1 (x^4+x)\,dx = \left[\int (x^4+x)\,dx \right] \Bigg|_{x=0}^{x=1}$$

$$= \left[\frac{1}{5}x^5 + \frac{1}{2}x^2 \right] \Bigg|_{x=0}^{x=1} = \left(\frac{1}{5}+\frac{1}{2}\right) - (0)$$

$$= \boxed{\frac{7}{10}}$$

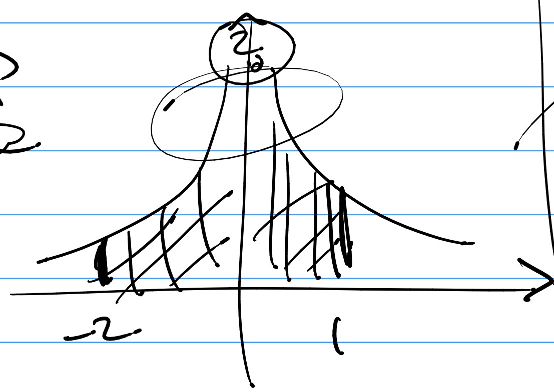
4.3 #49

$$\boxed{\int_{-2}^{1} x^{-4}\,dx} = \left.-\frac{1}{3}x^{-3}\right|_{x=-2}^{x=1} = \left(-\frac{1}{3}\right) - \left(-\frac{1}{3}(-2)^{-3}\right)$$

$$= -\frac{8}{24} - \frac{1}{3\cdot 8}$$

$$= -\frac{9}{24} = -\frac{3}{8}$$

$\frac{1}{x^4}$ → non-jump discon. @ $x=0$



$$\int f(x)\,dx = \boxed{\text{AntiDerivative}}$$

$$\int_a^b f(x)\,dx = \text{area under } f(x) \text{ over } [a,b]$$

b/c Fund. thm $\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]\Big|_{x=a}^{x=b}$

AntiDerivatives

① Table of known Derivatives → antiDerivatives

② Algebra / Trig

4.5 Study

$$D_x[f(g(x))] = f'(g(x))\,g'(x)$$

ex $D_x[\sin(x^2+1)] = \cos(x^2+1)\,(2x)$

ex $D_x[(x^2+x)^{1/2}] = \frac{1}{2}(x^2+x)^{-1/2}(2x+1)$

1st example $\int \cos(x^2+1)\cdot 2x\,dx = \sin(x^2+1)+C$

2nd example $\int \frac{1}{2}(x^2+x)^{-1/2}(2x+1)\,dx = \sqrt{x^2+x}+C$

Chain Rule $D_x[f(g(x))] = f'(g(x))\cdot g'(x)$

$$\int f'(g(x))\cdot g'(x)\,dx = f(g(x))+C$$

Tech Substitution Method

$$\int f(\underbrace{g(x)}_{u})\underbrace{g'(x)dx}_{du} = \int f(u)\,du$$

let $u = g(x)$
$du = g'(x)dx$

$= F(u)+C$

$= F(g(x))+C$

where F is f's antiderivative

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ex $\int \cos(x^2+x)(2x+1)\,dx = \int \cos(u)\,du$

let $u = x^2+x$
$du = (2x+1)dx$

$= \sin(u)+C$

$= \boxed{\sin(x^2+x)+C}$

(Ex) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \int \sin(\overset{u}{\sqrt{x}}) \frac{1}{\sqrt{x}}\,dx = 2\int \sin(u)\,du$

 $\frac{1}{\sqrt{x}}\,dx$ = $2du$

let $\boxed{\begin{array}{l} u=\sqrt{x} \\ du=\frac{1}{2\sqrt{x}}\,dx \end{array}} \to 2du = \frac{1}{\sqrt{x}}\,dx$

$= 2\int \sin(u)\,du = -2\cos(u) + C$

$= \boxed{-2\cos(\sqrt{x}) + C}$

(Ex) $\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du = \int (u-2)u^{1/2}\,du$

let $\boxed{\begin{array}{l} u=x+2 \\ du=dx \end{array}} \leftarrow x=u-2$

$= \int \left(u^{3/2} - 2u^{1/2}\right)du$

$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$

$= \boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$

(ex) $\int x^2\sqrt{x+2}\,dx = \int (u-2)^2\sqrt{u}\,du$

let $\boxed{\begin{array}{l} u=x+2 \\ du=dx \end{array}} \to x=u-2$

$= \int (u^2-4u+4)u^{1/2}\,du$

= etc.

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ex $\displaystyle\int_0^1 (3t-1)^{50}\,dt = \left[\int (3t-1)^{50}\,dt\right]\Bigg|_{t=0}^{t=1}$

let $\boxed{\begin{aligned} u &= 3t-1 \\ du &= 3\,dt \end{aligned}}$

$$= \left[\frac{1}{3}\int u^{50}\,du\right]\Bigg|_{t=0}^{t=1} = \frac{1}{153}u^{51}\Bigg|_{t=0}^{t=1}$$

$$= \frac{1}{153}(3t-1)^{51}\Bigg|_{t=0}^{t=1} = \left(\frac{1}{153}2^{51}\right) - \left(\frac{1}{153}(-1)^{51}\right)$$

$$= \boxed{\frac{1}{153}\left(2^{51}+1\right)}$$

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ex $\displaystyle\int_{t=0}^{t=1} (3t-1)^{50}\,dt = \boxed{\frac{1}{3}\int_{u=-1}^{u=2} u^{50}\,du} = \frac{1}{153}u^{51}\Bigg|_{u=-1}^{u=2}$

let $u = 3t-1$ $\quad t=0 \to u=-1$

$du = 3\,dt$ $\quad t=1 \to u=2$

$$= \left(\frac{1}{153}2^{51}\right) - \left(\frac{1}{153}(-1)^{51}\right)$$

$$= \boxed{\frac{1}{153}\left(2^{51}+1\right)}$$

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ex $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^4} = \int_{u=1}^{u=2} \frac{2\overset{u-1}{\sqrt{x}}\,du}{u^4} = \int_1^2 \frac{2(u-1)}{u^4}\,du$

let $\boxed{\begin{aligned} u &= 1+\sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}\,dx \end{aligned}}$ $\quad \begin{aligned} x=0 &\to u=1 \\ x=1 &\to u=2 \end{aligned}$

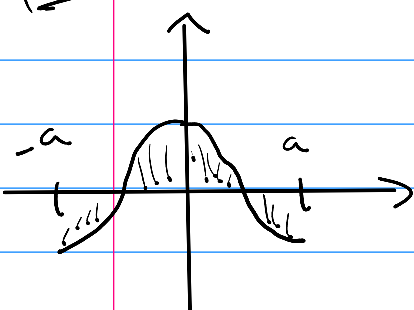
$$= 2\int_1^2 \frac{u-1}{u^4}\,du = 2\int_1^2 \left(u^{-3}-u^{-4}\right)du$$

$$= 2\left(-\frac{1}{2}u^{-2}+\frac{1}{3}u^{-3}\right)\Bigg|_{u=1}^{u=2} = \left[2\left(-\frac{1}{8}+\frac{1}{24}\right)\right]-\left[2\left(-\frac{1}{2}+\frac{1}{3}\right)\right]$$

$$= \left(-\frac{3}{12}+\frac{1}{12}\right)-\left(-\frac{3}{3}+\frac{2}{3}\right)$$

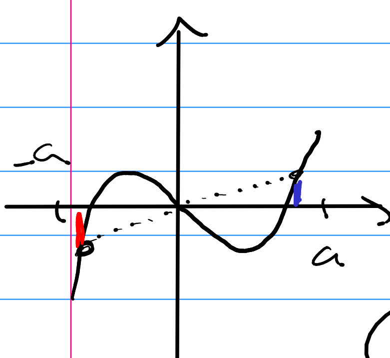
$$= \left(-\frac{1}{6}\right)+\frac{2}{6} = \boxed{\frac{1}{6}}$$

$\boxed{\text{Note:}}$ ① f is an even function (y-axis sym)



$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

② f is odd function (origin sym)



$$\int_{-a}^{a} f(x)\,dx = 0$$

ex $\displaystyle\int_{-100}^{100} \sin(x)\,dx = 0$

ex $\displaystyle\int_{-\pi}^{\pi} \left(\sin(x)+x^3-x^5\right)dx = 0$

ex $\displaystyle\int_{-\pi/2}^{\pi/2} \cos(x) + \sin(x)\, dx$

$$= \int_{-\pi/2}^{\pi/2} \cos(x)\,dx + \int_{-\pi/2}^{\pi/2} \sin(x)\,dx$$

$$= 2\int_0^{\pi/2} \cos x\,dx + 0$$

$$= 2\sin x \Big|_{x=0}^{x=\pi/2} = 2(1) - 2(0) = \boxed{2}$$

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ex $\displaystyle\int_0^1 x\sqrt{1-x^4}\,dx = \int_{u=1}^{u=0} \sqrt{u} \; -\frac{1}{4x^2}\,du$

let $u = 1-x^4$

$du = -4x^3\,dx$ $\qquad -\frac{1}{4x^2}\,du = x\,dx$

$x^2 = \sqrt{1-u}$

$\to -\frac{1}{4}\displaystyle\int_{u=1}^{u=0} \frac{\sqrt{u}}{\sqrt{1-u}}\,du$?☹ (try something else?)

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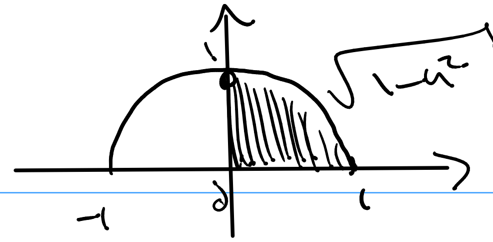
try 2 again.

$$\int_0^1 x\sqrt{1-x^4}\,dx = \frac{1}{2}\int_{u=0}^{u=1} \sqrt{1-u^2}\,du$$

let $u = x^2$

$du = 2x\,dx$

$$A = \frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du$$



$$= \frac{1}{2}\left(\frac{1}{4}\pi\right) = \boxed{\pi/8}$$