Math 243

Q's/ Midpt approx.

$$\int_1^3 \left(\cos(x) + x^3\right) dx \qquad n = 5$$

$$dx = \frac{3-1}{5} = \frac{2}{5} = .4$$

$y =$ f(1.2) f(1.6) f(2.0) f(2.4) f(2.8)

x = 1 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3

y = 2.0904 4.0668 7.5839 13.0866 21.0098

$$\text{Area} \approx 0.4\left(2.0904 + 4.0668 + 7.5839 + 13.0866 + 21.0098\right)$$

$$\text{Area} \approx \boxed{19.135}$$

$$\int_1^3 \cos x + x^3\, dx = \sin x + \frac{1}{4}x^4 \Big|_1^3$$

$$= \left(\sin(3) + \frac{1}{4}3^4\right) - \left(\sin(1) + \frac{1}{4}\right)$$

$$\approx \boxed{19.3}$$

$$\int_1^3 \left(\cos(x) + x^3\right) dx, \quad n = 5, \quad dx = \frac{3-1}{5} = \frac{2}{5} = .4$$

$y = f(1) \quad f(1.4) \quad f(1.8) \quad f(2.2) \quad f(2.6) \quad f(3)$

$x = 1 \quad 1.4 \quad 1.8 \quad 2.2 \quad 2.6 \quad 3$

$y =$ 1.5403 2.9140 5.6048 10.0595 16.7191 26.0100

$L_5 = 0.4($ 1.5403 + 2.9140 + 5.6048 + 10.0595 + 16.7191 $)$

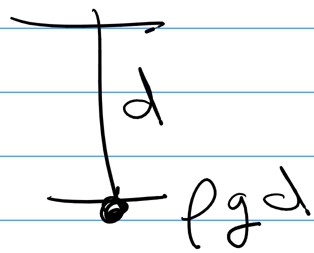
$R_5 = 0.4($ 2.9140 + 5.6048 + 10.0595 + 16.7191 + 26.0100 $)$

$T_5 = \frac{0.4}{2}($ 1.5403 + 2*2.9140 + 2*5.6048 + 2*10.0595 + 2*16.7191 + 26.0100 $)$

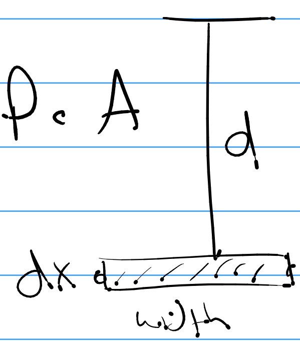
S_5 ← not even! → can not do simpson's rule!

8.3 Hydrostatic Pressure Force

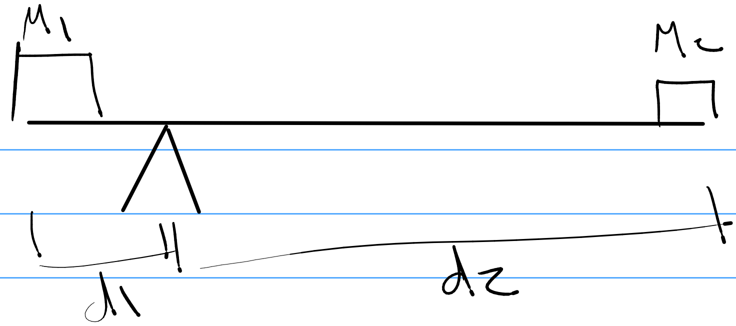
Pressure



Force = P · A

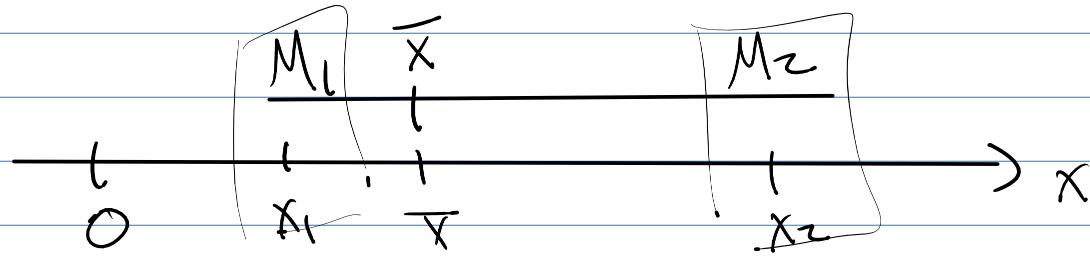


$$F = \int_a^b (\rho g \text{ depth})(\text{width})\, dx$$



$$m_1 d_1 = m_2 d_2$$

$\boxed{1D}$



$$m_1(\bar{x} - x_1) = m_2(x_2 - \bar{x})$$

$$m_1\bar{x} - m_1 x_1 = m_2 x_2 - m_2\bar{x}$$

$$m_1\bar{x} + m_2\bar{x} = m_1 x_1 + m_2 x_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

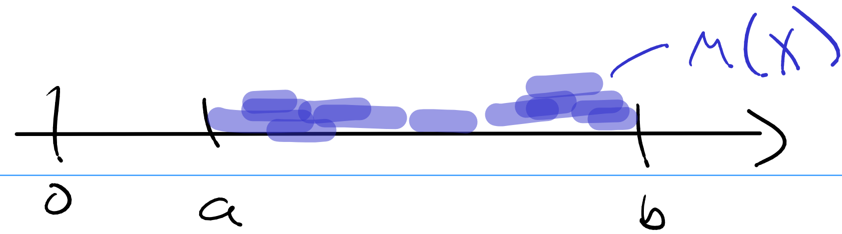
- Many mass objects $m_1, m_2, \dots, m_n$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_2} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

Moment according to the origin.

$$\bar{x} = \frac{\text{Moment according to origin}}{\text{Total Mass}}$$

1D

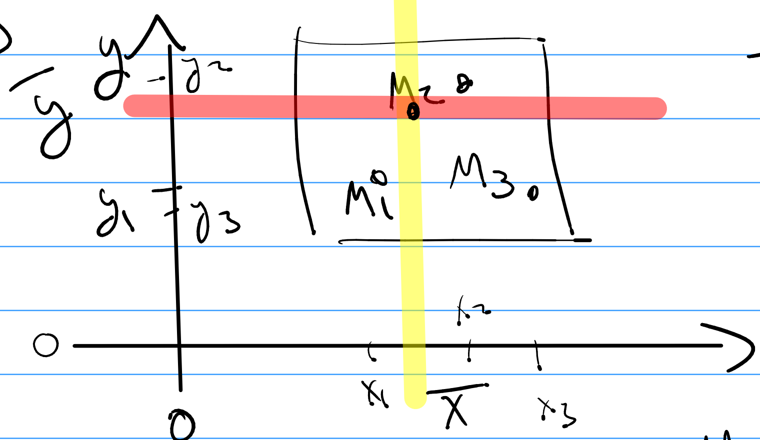


$$\text{Moment} = \int_a^b m(x)\cdot x\,dx$$

$$\text{Mass} = \int_a^b m(x)\,dx$$

$$\bar{x} = \frac{\int_a^b m(x)\cdot x\,dx}{\int_a^b m(x)\,dx}$$ Center of mass

2D



$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

x=0

x=0 y-axis

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\text{Moment from (y-axis)}}{\text{total mass}} = \frac{M_y}{M}$$

y=0

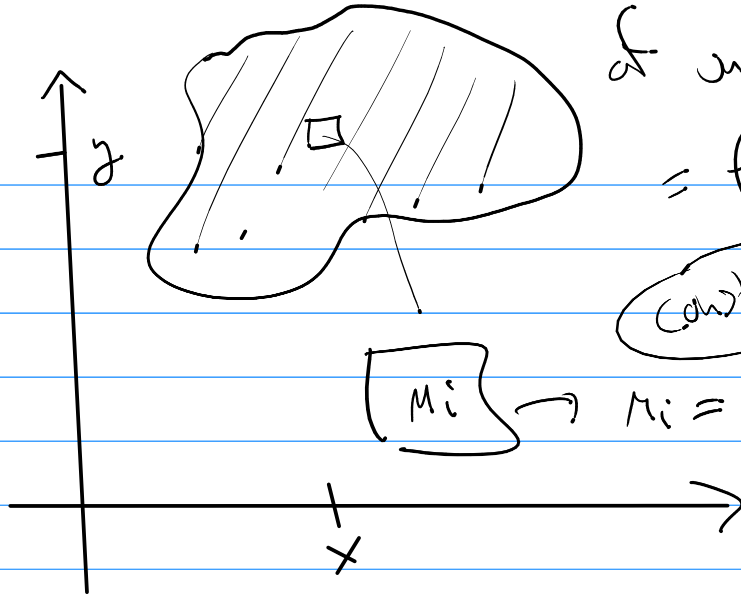
x-axis y=0

$$\bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} = \frac{\text{Moment from (x-axis)}}{\text{total mass}} = \frac{M_x}{M}$$

$\boxed{2D}$ Lamina

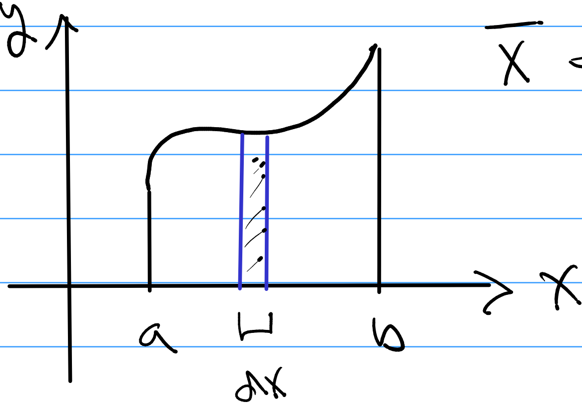
of uniform density $= \rho \ \frac{\text{mass}}{\text{unit}^2}$

constant



$\boxed{M_i} \rightarrow M_i = \rho(\text{area})$

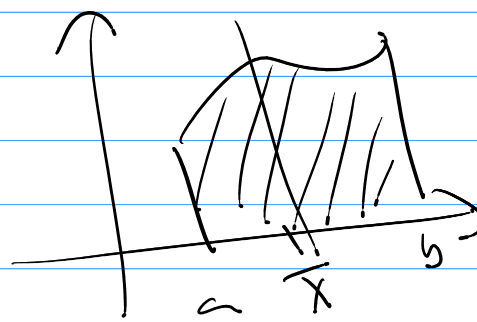
$\boxed{ex}$

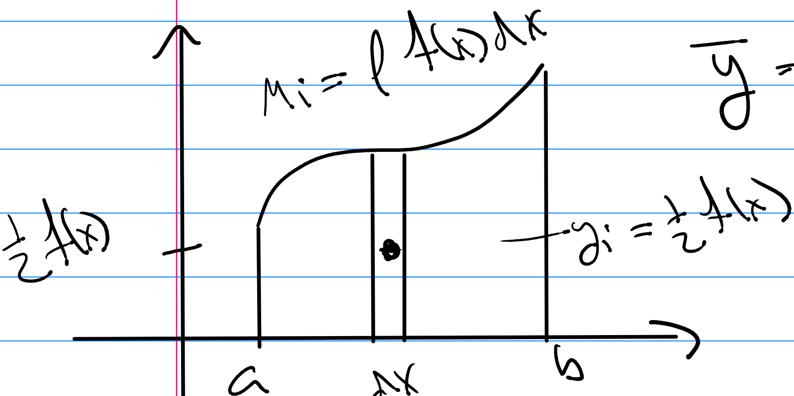


$$\bar{X} = \frac{M_{y\text{-axis}}}{M} = \frac{\int_a^b x(\overbrace{\rho f(x)dx}^{M_i})}{\int_a^b \rho f(x)dx}$$

$$\text{So } \bar{x} = \frac{\rho \int_a^b x f(x)dx}{\rho \int_a^b f(x)dx} = \frac{\int_a^b x f(x)dx}{\int_a^b f(x)dx}$$

$$\rightarrow \bar{X} = \frac{\int_a^b x f(x)dx}{\int_a^b f(x)dx}$$



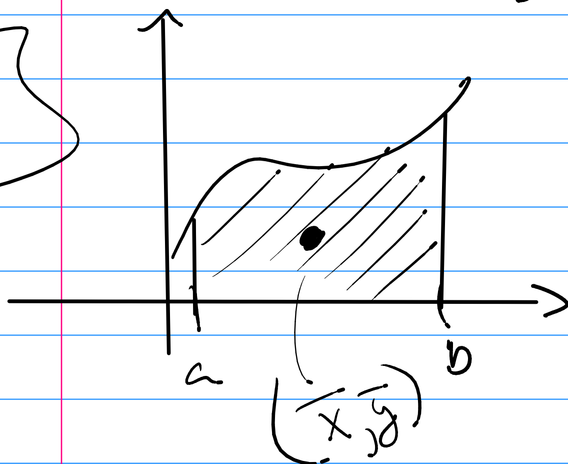


$$\bar{y} = \frac{\text{Moment from x-axis}}{\text{Mass}} = \frac{\sum M_i y_i}{\text{Mass}}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2}\rho (f(x))^2 dx}{\int_a^b \rho f(x)dx}$$

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f(x))^2\,dx}{\int_a^b f(x)\,dx}$$

$\int_D$



$$\bar{x} = \int_a^b x f(x)\,dx \Big/ \int_a^b f(x)\,dx$$

$$\bar{y} = \frac{1}{2}\int_a^b (f(x))^2\,dx \Big/ \int_a^b f(x)\,dx$$

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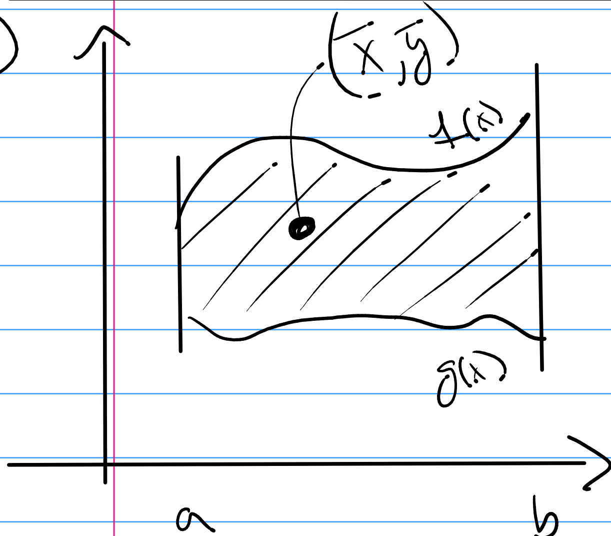
ex



$$\bar{x} = \frac{\int_a^b (x f(x) - x g(x))\,dx}{\int_a^b (f(x) - g(x))\,dx}$$

$$\bar{y} = \frac{\frac{1}{2}\int_a^b (f^2 - g^2)\,dx}{\int_a^b (f - g)\,dx}$$

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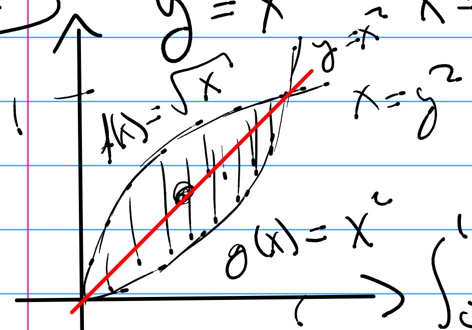
ex

$y = x^2 \quad x = y^2$

guess $(\bar{x}, \bar{y}) = (0.5, 0.5)$



$$\int_0^1 (\sqrt{x} - x^2)\,dx = \frac{2}{3}x^{3/2} - \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{3}$$

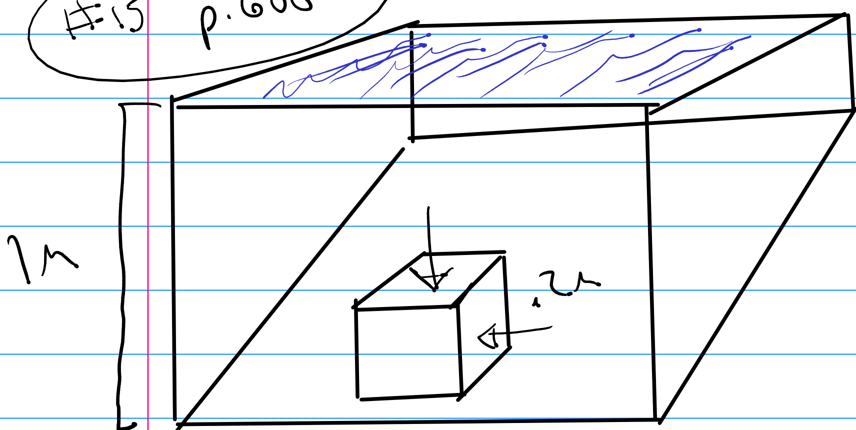
$$\int_0^1 x(\sqrt{x}) - x(x^2)\,dx = \frac{2}{5}x^{5/2} - \frac{1}{4}x^4 \Big|_0^1 = \frac{3}{20}$$

$$\int_0^1 \frac{1}{2}(\sqrt{x})^2 - \frac{1}{2}(x^2)^2\,dx = \frac{1}{4}x^2 - \frac{1}{10}x^5 \Big|_0^1 = \frac{3}{20}$$

$\bar{x} = \dfrac{3/20}{1/3} = 9/20 \qquad \bar{y} = \dfrac{3/20}{1/3} = 9/20$

$$(\bar{x}, \bar{y}) = \boxed{\left(9/20,\ 9/20\right)}$$

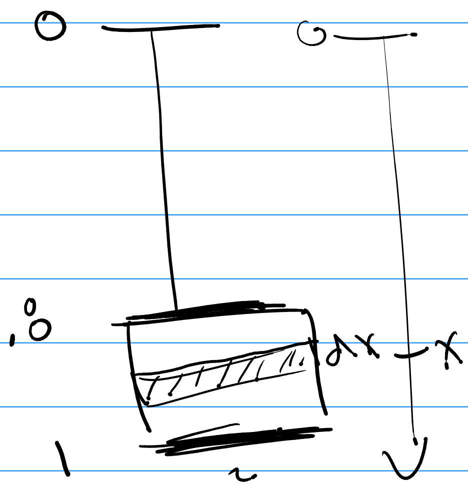
#15 p.606



top = (pressure)(area)

$$\text{top} = \boxed{\rho g(.8)(.2)^2}$$

Side



$$\int_{.8}^{1} (\rho g x)(.2\,dx)$$

$$= .2\rho g \int_{.8}^{1} x\,dx$$

$$= .2\rho g\left(\tfrac{1}{2}x^2\right)\Big|_{.8}^{1}$$

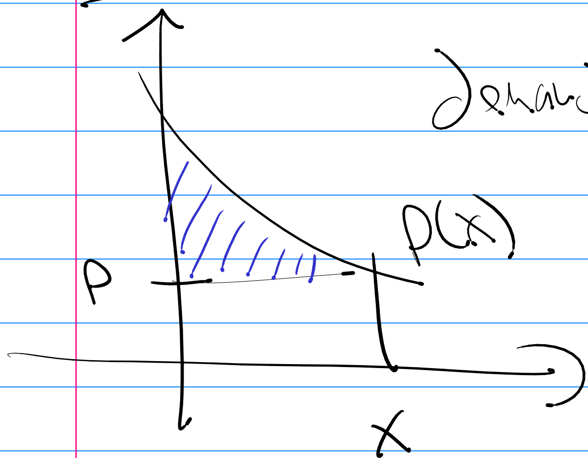
$$= \boxed{.2\rho g\left(\tfrac{1}{2} - \tfrac{1}{2}(.8)^2\right)}$$

$$= .1\rho g(1 - .64) = .1\rho g(.36)$$

$$= \boxed{.036\rho g}$$

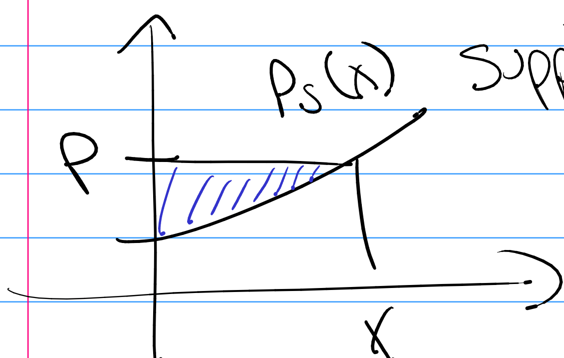
Business / Econ

demand function



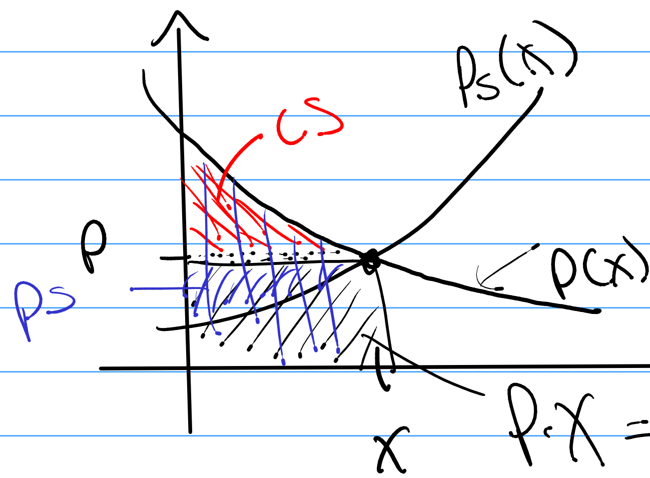
$$CS = \int_0^X \left(p(x) - P\right)\,dx$$

Supply function



$$PS = \int_0^X \left(P - p_s(x)\right)\,dx$$

Equilibrium

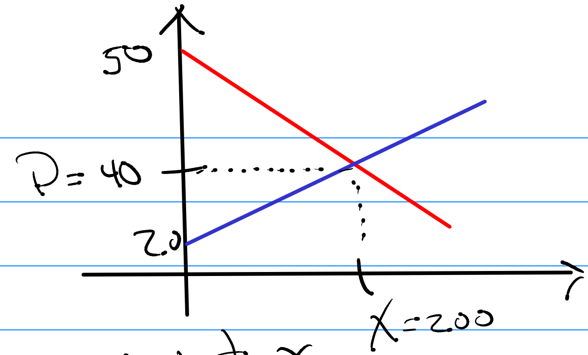


$P \cdot X = \$$ real money

ex

$$p(x) = 50 - \frac{1}{20}x$$

$$p_s(x) = 20 + \frac{1}{10}x$$



Equilibrium

$$50 - \frac{1}{20}x = 20 + \frac{1}{10}x$$

$$30 = \frac{3}{20}x$$

$$x = 200 \qquad P = 40$$

$$CS = \int_0^{200} \left(\left(50 - \frac{1}{20}x\right) - 40\right)dx = \int_0^{200} 10 - \frac{1}{20}x\,dx$$

$$CS = 10x - \frac{1}{40}x^2 \Big|_0^{200} = 2000 - 1000 = \boxed{1000}$$

$$PS = \int_0^{200} (40) - \left(20 + \frac{1}{10}x\right)dx = \int_0^{200} 20 - \frac{1}{10}x\,dx$$

$$= 20x - \frac{1}{20}x^2 \Big|_0^{200} = 4000 - 2000 = \boxed{2000}$$

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ex

$$p(x) = \frac{900{,}000\, e^{-x/500}}{x + 20{,}000} \qquad p(x) = 16$$

Solve?

$$16 = \frac{900{,}000\, e^{-x/500}}{x + 20{,}000}$$

$$\to 16x + 320{,}000 = 900{,}000\, e^{-x/500}$$

$$\to \ln(16x + 320{,}000) = \ln(900{,}000) - x/500$$

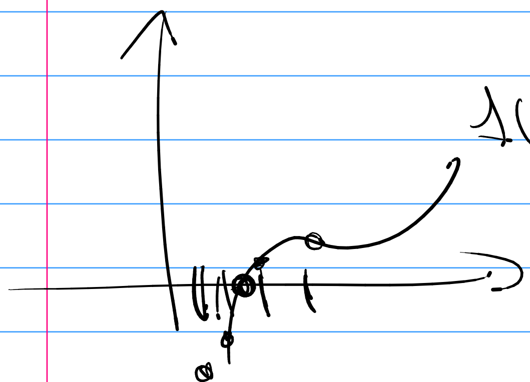
?

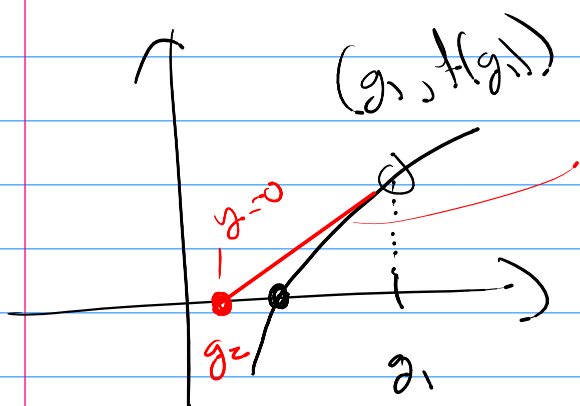
$$16 = \frac{800{,}000\, e^{-x/500}}{x+20{,}000} \to \boxed{\frac{800000\, e^{-x/500}}{x+20000} - 16} = 0$$

Solve $f(x) = 0$ **but** can't do it by hand!

try #1 bisection method?

try #2 Newton's method?





$$y - f(g_1) = f'(g_1)(x - g_1)$$

let $y = 0$ solve for x

$$g_2 = g_1 - \frac{f(g_1)}{f'(g_1)}$$

$$g_n = g_{n-1} - \frac{f(g_{n-1})}{f'(g_{n-1})}$$

$g_1 \to g_2 \to g_3 \to \cdots$ until $f(g_n) \approx 0$

$$f(x) = \frac{800000\, e^{-x/500}}{x + 20000} - 16$$

$$f'(x) = \frac{\left(800000\, e^{-x/500}\right)\left(-\frac{1}{500}\right)(x+20000) - \left(800000\, e^{-x/500}\right)(1)}{(x+20000)^2}$$

$$f'(x) = 800000\, e^{-x/500}\, \frac{\left(-\frac{x}{500} - 41\right)}{(x+20000)^2}$$

$$f(x) = \frac{800000\, e^{-x/500}}{x+20000} - 16$$

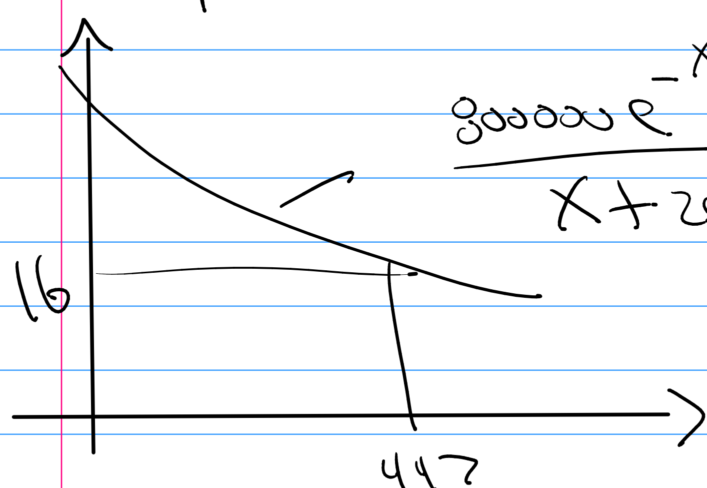
$$g_n = g_{n-1} - \frac{f(g_{n-1})}{f'(g_{n-1})}$$

g's
440
447.04
447.09

$447.09 \rightarrow f(447.09) \approx 10^{-15}$

$\boxed{X = 447.09}$ demand $\qquad P = \boxed{\$16 \text{ price}}$

Say $X = 447$ $\qquad P = \$16$



$$\frac{800000\, e^{-x/500}}{X+20000} = P(x)$$