Math 293

Exam 2

(1) $\int_1^2 x^2 \ln x\,dx = \frac{1}{3}x^3\ln x\Big|_1^2 - \frac{1}{3}\int_1^2 x^2\,dx = \underline{\text{etc}}$

$f(x) = \ln x \to f'(x) = 1/x$

$g'(x) = x^2 \to g(x) = \frac{1}{3}x^3$

(2) $\int e^{\overset{u}{\sin x}} \overset{u}{\sin x}\underbrace{\cos x\,dx}_{du} = \int u e^u\,du = ue^u - \int e^u\,du$

let $u = \sin x$ $\quad f(u) = u \to f'(u) = 1$

$du = \cos x\,dx$ $\quad g'(u) = e^u \to g(u) = e^u$

$\to ue^u - e^u + C = \underline{\text{etc}}$

(3) $\int_0^{\pi/2} \sin^7 x \cos^5 x\,dx = \int_0^{\pi/2} \sin^7 x \cos^4 x \cos x\,dx$

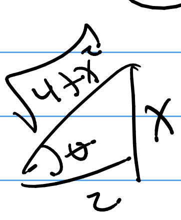
let $u = \sin x$ $du = \cos x\,dx$ $\quad \int_0^1 u^7(1-u^2)^2\,du$

$= \int_0^1 u^7(1-2u^2+u^4)\,du = \underline{\text{etc}}$

(4) $\int \tan^3 x \sec^4 x\,dx = \int \tan^3 x \sec^2 x(\sec^2 x\,dx)$

let $u = \tan x$ $du = \sec^2 x$ $\quad \to \int u^3(1+u^2)\,du = \underline{\text{etc}}$

(5) $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = \int \frac{1}{4\tan^2\theta\sqrt{4+4\tan^2\theta}}\,2\sec^2\theta\,d\theta$



let $x = 2\tan\theta$ $\quad \to \frac{1}{\cancel{8}4}\int \frac{1}{\tan^2\theta\,\cancel{\sec\theta}}\,\cancel{2}\sec^2\theta\,d\theta$

$dx = 2\sec^2\theta\,d\theta$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^{\cancel{2}}\theta}}\,d\theta$$

$$= \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{1}{4}\int u^{-2}\,du = -\frac{1}{4}u^{-1} + C$$

let $u = \sin\theta$ $\quad du = \cos\theta\, d\theta$ $\qquad = \frac{-1}{4\sin\theta} + C$

$$= \boxed{-\frac{1}{4}\,\frac{\sqrt{4+x^2}}{x} + C}$$

⑥ $\displaystyle\int \frac{x^2+x+1}{x(x^2+1)}\,dx \qquad \frac{C_1}{x} + \frac{C_2x + C_3}{x^2+1}$

$$x^2+x+1 = C_1(x^2+1) + (C_2x + C_3)x$$

x^2: $1 = C_1 + C_2 \rightarrow C_2 = 0$

x: $1 = C_3$

const: $1 = C_1$

$$\int \frac{1}{x} + \frac{1}{x^2+1}\,dx$$

$$= \ln|x| + \tan^{-1}x + C$$

⑦ $\displaystyle\int \frac{\tan^3 x}{\cos^3 x}\,dx \rightarrow \int \tan^3 x \sec^3 x\,dx$ let $u = \sec x$

etc.

or

$$\int \frac{\sin^2 x\,\sin x}{\cos^6 x}\,dx$$

let $u = \cos x$

$du = -\sin x\,dx$

$$-\int \frac{1-u^2}{u^6}\,du = \int \frac{u^2}{u^6} - \frac{1}{u^6}\,du$$

$$= \int u^{-4} - u^{-6}\,du = \text{etc}$$

(8) $\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$

$f(x) = \sin x \to f'(x) = \cos x$ $\quad$ $f(x) = \cos x \to f'(x) = -\sin x$

$g'(x) = e^x \to g(x) = e^x$ $\quad$ $g'(x) = e^x \to g(x) = e^x$

$= e^x \sin x - e^x \cos x - \int e^x \sin x\, dx$

$\to \int e^x \sin x\, dx = \frac{1}{2} e^x (\sin x - \cos x) + C$

(9) $\int x^2 \sqrt{5-2x^2}\, dx$ $\quad a = -2 \quad b = 5$

(#54) (a neg) $\int \underbrace{\sqrt{ax^2+b}}_{3/2}\, x^2\, dx$

$= \frac{x}{4a}(ax^2+b) - \frac{bx}{8a}\sqrt{ax^2+b}\; \boxed{-\frac{b^2}{8a\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{b}}\right)}$

(10) $\int \frac{1}{\sqrt{e^{2x}-3}}\, dx = \int \frac{1}{u\sqrt{u^2-3}}\, du = \frac{\text{etc}}{2}$

$e^{2x} = (e^x)^2$

$u = e^x$

$du = e^x dx \to dx = \frac{1}{u} du$

(11) $\int_0^1 \frac{1}{\sqrt{e^{2x}+3}}\, dx$ $\quad n = 4 \quad dx = 0.25$

$\frac{1}{\sqrt{4}} = y_0 \quad \frac{1}{\sqrt{e^{.5}+3}} = y_1 \quad y_2 \quad y_3 \quad y_4$

$x = \quad 0 \quad .25 \quad .5 \quad .75 \quad 1$

$S_4 =$

$L_4 = 0.25\left(\frac{1}{\sqrt{4}} + y_1 + y_2 + y_3\right)$ etc

(12) $$\int_1^\infty \frac{1}{(x+1)^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{(x+1)^2}\,dx$$

let $u = x+1$, $du = dx$

$$\lim_{b\to\infty} \int_{x=1}^{x=b} u^{-2}\,du = \lim_{b\to\infty}\left(-\cancel{\frac{1}{b+1}}^{0} + \frac{1}{2}\right)$$

converges

(13) $$AL = \int_a^b \sqrt{1+(f')^2}\,dx$$

$$= \int_0^3 \sqrt{1+(e^x)^2}\,dx$$

(14) $$y = x^2 - \frac{1}{8}\ln x \qquad y' = 2x - \frac{1}{8x}$$

$$\sqrt{1+(y')^2} = \sqrt{4x^2 + \frac{1}{2} + \frac{1}{64x^2}}$$

$$= \sqrt{\left(2x + \frac{1}{8x}\right)^2} = 2x + \frac{1}{8x}$$

$$\to AL = \int_1^3 2x + \frac{1}{8x}\,dx = x^2 + \frac{1}{8}\ln|x|\Big|_1^3$$

$$= \left[9 + \frac{1}{8}\ln 3\right] - [1] = \boxed{8 + \frac{1}{8}\ln 3}$$

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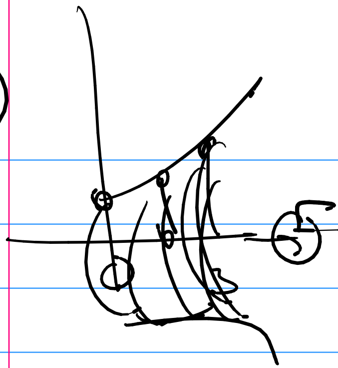

$$\boxed{\int_0^3 2\pi e^x \sqrt{1+(e^x)^2}\, dx}$$

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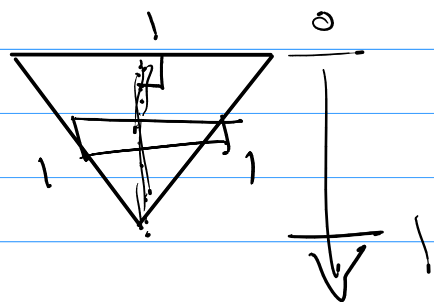
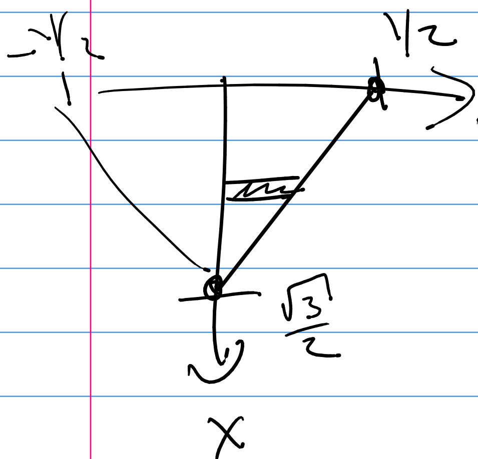


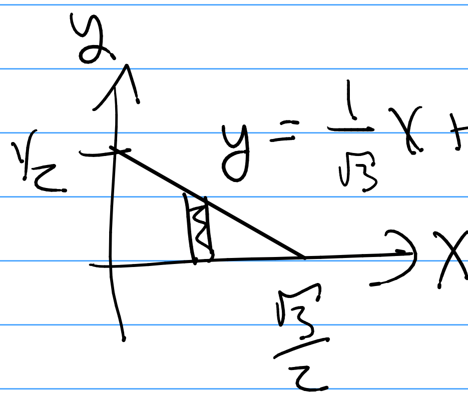
$$10g \int_0^{\sqrt{3}/2} x\,(\text{width})\,dx$$

$$\rightarrow 10g \int_0^{\sqrt{3}/2} x\left(\tfrac{2}{\sqrt{3}}x+1\right)dx$$

$\sqrt{\ldots 3}$



width $= 2y$



$$y = \frac{1}{\sqrt{3}}x + \frac{1}{2}$$

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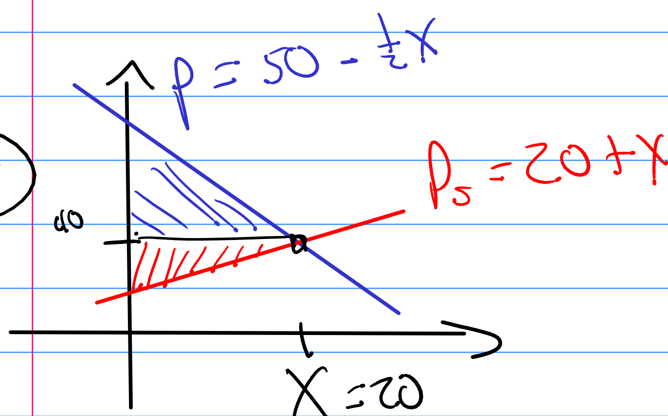


$$50 - \tfrac{1}{2}x = 20 + x$$

$$30 = \tfrac{3}{2}x$$

$$20 = x$$

$$\rightarrow P(20) = 40$$

$$CS = \int_0^{20} \left(50 - \tfrac{1}{2}x\right) - 40\, dx = \ldots$$

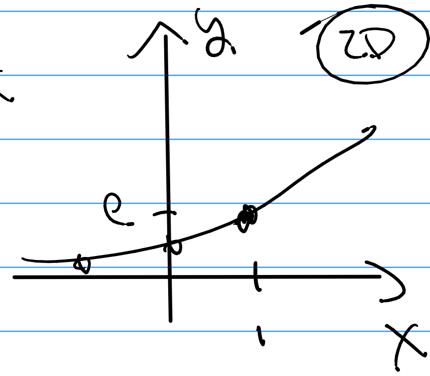
$$PS = \int_0^{20} 20 - (20 + x)\, dx = \ldots$$

Ch 10 Parametric Eqns and Polar Coord.

10.1

$f(x) = e^x$

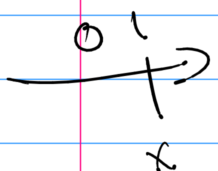
2D

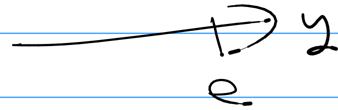


x	y
0	1
1	e
-1	$1/e$

y ← dependent variable

x ← independent variable





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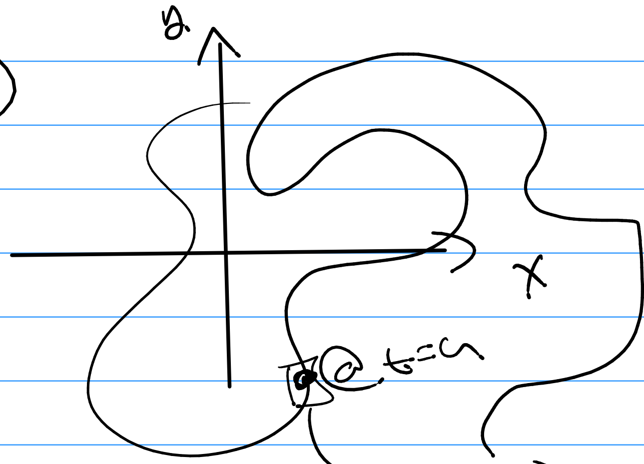
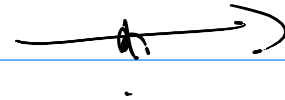




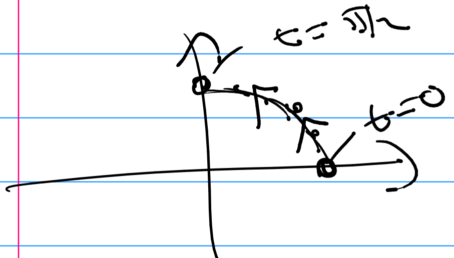
t — parameter

$$\left.\begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned}\right\} \text{ parametric equations}$$

t	x, y
0	$(x(0), y(0))$

ex: $x(t) = \cos(t)$ $\quad y(t) = \sin(t)$

$0 \le t \le 2\pi$



t	x, y
0	$(1, 0)$
$\pi/2$	$(0, 1)$

Notice: $x^2 = \cos^2 t$ $y^2 = \sin^2 t$

$$\rightarrow x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

So $x^2 + y^2 = 1$