Math 243

~~Q's~~ Final, Monday May 8th @ 3pm

Exams 1–4 ~~take~~ 4 probs / exam = 16 probs

@ 10 pts each

150 pts = 100%

→ modify the probs . .

ex. exam 1 radius, center (eqn's of spheres)

$$x^2+y^2+z^2-2x+4y=11$$

Final: ? $x^2+y^2+z^2-3x+2z-y=12$

?, center is the intersection of

$$2x+y=3z, \quad z=4$$

$$y=x-7$$

it's radius is $\sqrt{3}$. → eqn?

Math 243 ... Final Review

Exam 1 (4 probs)

1) Find an equation of the sphere with center (3,8,1) and passing through the point (2,6,4).

2) Find the center and radius of the sphere: $x^2 + y^2 + z^2 - 2x + 4y = 11$

3) For $\boldsymbol{a} = \langle 1, 2, 3\rangle$ and $\boldsymbol{b} = \langle -2, 1, 2\rangle$ find ...

a) $|2\boldsymbol{a} - \boldsymbol{b}|$ c) $\vec{a} \times \vec{b}$

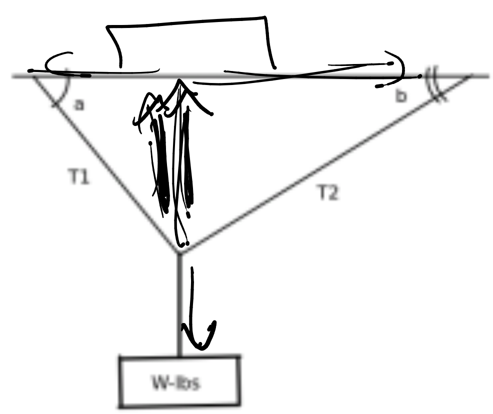
b) $\boldsymbol{a} \cdot \boldsymbol{b}$

ops on vectors

4) If you were given two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, what would you do to determine if they were parallel? If they were perpendicular?

5) Find a nonzero vector orthogonal to the plane through the points (1,0,1), (-1,1,3), and (4,2,2).

No 6) A W-lb weight hangs from two wires as shown. Find magnitude of the tensions, T1 and T2, in both wires.



7) A tow truck drags a car along a road. The chain makes an angle of 32^o with the road and the tension in the chain is 1200 Newtons. How much work is done by the truck if it is pulled 1 Kilometer?

No 8) A bolt is tightened by applying a 10-N force to the end of 20cm wrench at an angle of 61^o. Find the magnitude of the torque about the center of the bolt.

No 9) You study a bacterium and find that a single cell divides into two cells every 15 minutes. Given that you start with an initial population of 10 cells ...

a) Find the growth rate (use hours for the unit of time).

b) Find an expression for the number of cells after t hours.

No 10) Find the value of the expression: $\tan(\sin^{-1}(2/3))$

? 11) Prove that $\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{\partial}{\partial x}[\sin^{-1}(x)] =$ or $\frac{d}{dx}[\cos^{-1}(x)] =$ etc

12) Perform the indicated problems (do not simplify expressions in your answers) ...

a) Find the derivative of $\ln(x)\sin^{-1}(2x+1)$

b) $\lim_{x\to 0}\frac{\arctan(x)}{\sin(x)}$

?o

13) Find ...

a)

$$\int \frac{1}{1+x^2} + \frac{1}{x} + \sin(x)\,dx$$

b)

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$$

Yes 14) Perform the indicated problems (do not simplify expressions in your answers) ...

a) Find the derivative of $\sinh(x)/(x^2+3x)$

b) $\lim_{x\to 0}\frac{\tanh(x)}{\tan(x)}$

No 15) Prove that $\frac{d}{dx}(\sinh^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$

? 16) Perform the indicated problems (do not simplify expressions in your answers) ...

Find the derivative of $e^{2x} + \ln(x^2) - \sin(x) + \tanh^{-1}(2x+1)$

? 17) Find ...

a)

$$\int \frac{1}{\sqrt{1+x^2}}\,dx$$

b)

$$\int \frac{1}{\sqrt{4+x^2}}\, dx$$

Exam 2

No 1) Use integration by parts to evaluate the given integral.

$$\int_1^2 x^2 \ln(x)\, dx$$

?

? 2) Use substitution and integration by parts to evaluate the given integral.

$$\int e^{\sin(t)} \sin(t)\cos(t)\, dt$$

? 3) Evaluate the given trigonometric integral.

$$\int_0^{\pi/2} \sin^7(x)\cos^5(x)\, dx$$

No 4) Evaluate the given trigonometic integral.

$$\int \tan^3(x)\sec^4(x)\, dx$$

No 5) Use trigonometric substitution to evaluate the given integral.

$$\int \frac{1}{x^2\sqrt{4+x^2}}\, dx$$

? 6) Use partial fractions to evaluate the given integral.

$$\int \frac{x^2+x+1}{x^3+x}\, dx$$

7) Evaluate the given integral.

Yes ← Yes, an Integral w/o being told what to do

$$\int \frac{\tan^3(x)}{\cos^3(x)}\, dx$$

at least one

8) Evaluate the given integral.

$$\int e^x \sin(x)\, dx$$

No 9) Use the table to evaluate the given integral.

$$\int x^2\sqrt{5-2x^2}\,dx$$

48 $\displaystyle\int \frac{x\,dx}{\sqrt{ax^2+b}} = \frac{1}{a}\sqrt{ax^2+b}.$

49 $\displaystyle\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} + \sqrt{b}\ln\frac{\sqrt{ax^2+b}-\sqrt{b}}{x}.$ (b pos.)

50 $\displaystyle\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} - \sqrt{-b}\tan^{-1}\frac{\sqrt{ax^2+b}}{\sqrt{-b}}.$ (b neg.)

51 $\displaystyle\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{b}}\ln\frac{\sqrt{ax^2+b}-\sqrt{b}}{x}.$ (b pos.)

52 $\displaystyle\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{-b}}\sec^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$ (b neg.)

53 $\displaystyle\int \sqrt{ax^2+b}\,x^2\,dx = \frac{x}{4a}(ax^2+b)^{\frac{3}{2}} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$ (a pos.)

54 $\displaystyle\int \sqrt{ax^2+b}\,x^2\,dx = \frac{x}{4a}(ax^2+b)^{\frac{3}{2}} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{-a}}\sin^{-1}\left(x\sqrt{\frac{-a}{b}}\right).$ (a neg.)

Yes 10) Use substitution and the above table to evaluate the given integral.

$$\int \frac{1}{\sqrt{e^{2x}-3}}\,dx$$

No 11) Setup the solutions for the left endpoint approximation and the simpson approximation for the given integral using 4 intervals, do not evaluate.

$$\int_0^1 \frac{1}{\sqrt{e^{2x}+3}}\,dx$$

? 12) Evaluate the given improper integral.

$$\int_1^\infty \frac{1}{(x-1)^2}\,dx$$

? 13) Setup, but do not integrate, an integral to find the arc length of the curve: $y=e^x$, $0\le x\le 3$.

? 14) Find the arc length of the curve: $y=x^2-\frac{1}{8}\ln(x)$ between $1\le x\le 3$.

? 15) Setup, but do not integrate, an integral to find the area of the surface generated by rotating the curve $y=e^x$, $0\le x\le 3$, about the x-axis.

know arc length, surface area formulas.

No 16) A trough is filled with a liquid of density 10 kg$/m^3$. The ends of the trough are equilateral triangles with sides 1m long and vertex at the bottom. Find the hydrostatic force on one end of the trough.

No 17) The demand curve is $p(x) = 50 - \frac{1}{2}x$ and the supply curve is $p_s(x) = 20 + x$. What is the quantity and price when the market is in equilibrium? Find the consumer surplus, producer surplus, and sketch an illustration of the curves and label the surpluses as areas.

Exam 3

No 1) Sketch the parametric curve by eliminating the parameter to find a Cartesian equation.

$$x = t^2 - 2t, \quad y = t + 1, \quad 0 \leqslant t \leqslant 4$$

No 2) Find dy/dx and d^2y/dx^2.

$$x = t^2 - 2t, \quad y = t + 1$$

Yes 3) Find the points on the curve where the tangent is horizontal or vertical. And determine where the curve is concave upward or downward.

$$x = t^2, \quad y = t^3 - 3t$$

4) Use the parametric equations of an ellipse, $x = 2\cos(t)$, $y = 3\sin(t)$, $0 \leqslant t \leqslant 2\pi$, to find the area that it encloses.

5) Find the exact length of the curve: $x = 1 + t^2$, $y = 1 + t^3$, $0 \leqslant t \leqslant 1$.

? 6) Find the surface area of a sphere by rotating $x = r\cos(t)$, $y = r\sin(t)$, $0 \leqslant t \leqslant \pi$ about the x-axis.

7) a) Convert $(2, \pi/3)$ from polar to Cartesian coordinates.

b) Give two polar representations for the Cartesian point (1,1).

8) Find the points on the curve $r = \cos(\theta)$ where the tangent is horizontal or vertical.

9) Identify the curve by finding a Cartesian equation for the curve $r = 4\sec(\theta)$.

No 10) Find the area of the shaded region.

No 11) Find the area of the region inside $r = 4\sin(\theta)$ and outside $r = 2$.

? 12) Find the exact length of the polar curve formed by the area of the region inside $r = 4\cos(\theta)$ and outside $r = 2$.

No 13) Sketch the conic $9x^2 - 4y^2 - 72x + 8y + 176 = 0$ and find its foci.

Conic

? 14) Find an equation for a ellipse with foci $(\pm 2, 0)$ and vertices $(\pm 5, 0)$.

eqn's of conics in cartesian.

? 15) Find the area enclosed by the hyperbola $x^2 / a^2 - y^2 / b^2 = 1$ and a vertical line through a focus.

No 16) Write a polar equation for a hyperbola with eccentricity 3 and directrix $x = 3$. Sketch it.

? 17) Find the eccentricity, identify the conic, give and equation for the directrix and sketch the conic $r = 5/(2 - 4\cos(\theta))$.

eqn's of conics in polar

Exam 4

? 1) Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $a_n = \sqrt{n+3} / \sqrt{n-4}$

b) $a_n = 3^n 7^{-n}$

c) $a_n = \ln(n) / \ln(2n)$

No 2) Show that the geometric series $a + ar + ar^2 + ar^3 + \ldots = a/(1-r)$ when $|r| < 1$.

? 3) Use the integral test to determine whether the series ...

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

is convergent or divergent.

Yes 4) Use the limit comparison test to determine whether the series ...

$$\sum_{n=1}^{\infty} \frac{n^2+n}{\sqrt{1+n^5}}$$

is convergent or divergent.

?

5) Use the root test to determine whether the series ...

$$\sum_{n=1}^{\infty}\left(\frac{-2}{n}\right)^n$$

is convergent or divergent.

6) Determine whether the series ...

$$\sum_{n=1}^{\infty}\frac{(-9)^n}{n\,10^{n+1}}$$

is absolutely convergent, conditionally convergent, or divergent.

7) Determine whether the series ...

$$\sum_{n=1}^{\infty}(-1)^n\frac{n^2-1}{n^2+1}$$

is convergent or divergent.

Yes

8) Find the radius of convergence and interval of convergence of the series.

Ratio test

$$\sum_{n=1}^{\infty}\frac{(3x-1)^n}{4^n\sqrt{n}}$$

?

9) Find a power series representation with interval of convergence for the function ...

$$f(x)=\frac{x}{2x^2+1}$$

... by using the power series $1/(1-x)=1+x+x^2+x^3+\ldots.$ for $|x|<1$

?

10) Find the Taylor series for $f(x)=1/x$ centered at $a=-3$.

No

11) Approximate $f(x)=x^{2/3}$ with a Taylor polynomial of degree $n=3$ centered at $a=1$. What do you know about the accuracy of your approximation on the interval $0.9\leqslant x\leqslant 1.1$?

No

12) A uniformly charged disk of radius R and surface charge density σ has an electric potential of V at a point d units away along the perpendicular central axis is ...

$$V=2\pi k_e\sigma\sqrt{d^2+R^2}-d$$

Show that for large d ...

$$V\approx\frac{\sigma k_e\pi R^2}{d}$$