Math 344

plotter (online) http://wims.unice.fr/wims/en_home.html

(try polyray)

$$Ax^2 + By^2 + Cz^2 = D$$

① D is positive $\wedge$ A, B, C are positive

→ ellipsoid

② D is positive $\wedge$ A, B, C exactly two neg

→ Hyperboloid of 2 sheets

③ D is positive $\wedge$ A, B, C exactly one neg

→ Hyperbolod of 1 sheet

④ D is zero $\wedge$ A, B, C exactly one neg
exactly two neg

(and)

→ $\underline{\underline{\text{Cones}}}$

$$Ax^2 + By^2 + Cz = 0$$

(C is positive)

① A, B neg $\rightarrow$ Elliptic Paraboloid

② A, B one pos one neg $\rightarrow$ Hyperbolic Paraboloid

$y = x^3 - x$ $\quad \mathbb{R}^2$

Shape recog?

$\sin(300x)$ $\quad \mathbb{R}^2$

$x^2 + y^2 + z^2 = 7$ in $\mathbb{R}^3$

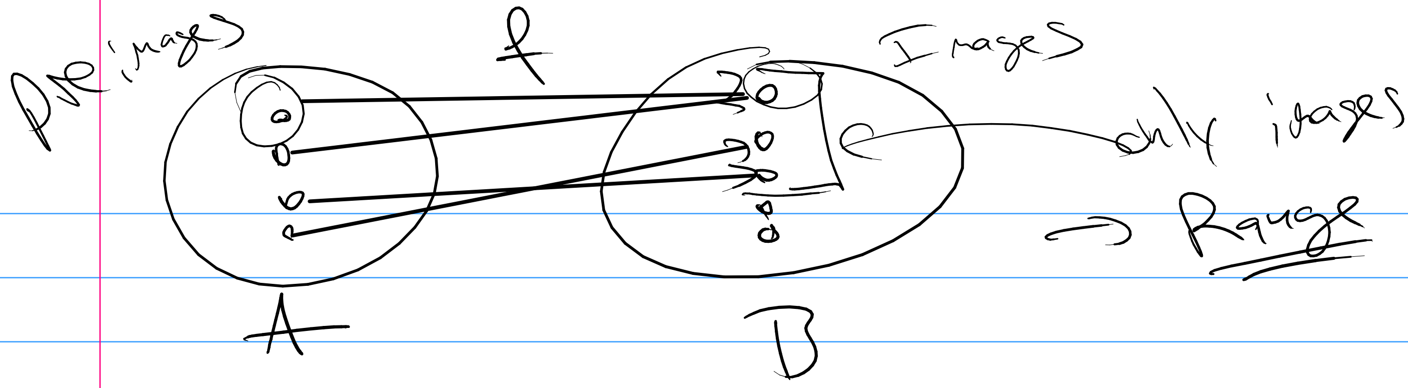
$y = x^3 + 1$ in $\mathbb{R}^3$

10.1 Vector Functions / Space Curves

Functions: (Rules map elements)

$$f : A \rightarrow B$$

domain $\quad$ codomain

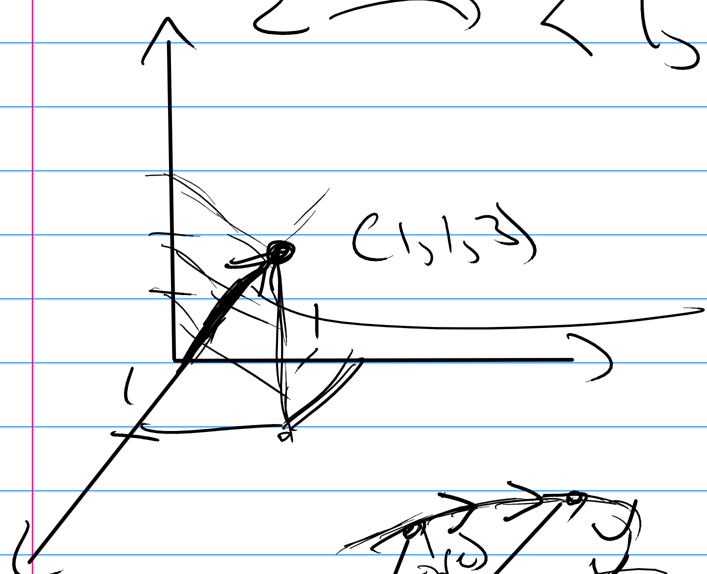


Calc 1, 2 $\quad f: \mathbb{R} \to \mathbb{R}$

$\boxed{10.7}$ Vector Functions

$$f: \mathbb{R} \to \left(\begin{matrix} \text{Set of} \\ \text{Vectors} \end{matrix} \right)$$

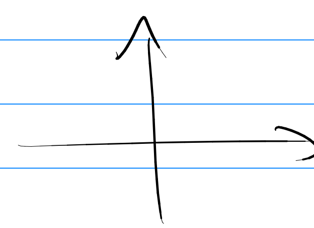
$$2 \xrightarrow{\vec{f}} \langle 1, 1, 3 \rangle$$



$\vec{f}(2) = \langle 1, 1, 3 \rangle$

ex

$\vec{r}(t)$



So $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

$x = f(t)$

$y = g(t)$ $\qquad t \in \mathbb{R}$

$z = h(t)$

element of $\qquad$ real numbers

t = parameter $\qquad (x, y, z)$ parametric curve

→ Parametric curve is traced by $\vec{r}(t)$ in direction of inc. t's.

Space Curve

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what to do with $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

① Evaluate

ex $\vec{r}(t) = \langle \sinh(t), \cos(t), e^t \rangle$

$\vec{r}(1) = \langle \sinh(1), \cos(1), e \rangle$

② $\lim_{t \to a} \vec{r}(t)$

$$\lim_{t\to a} \vec{r}(t) = \lim_{t\to a} \langle f(t), g(t), h(t) \rangle$$

$$= \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \rangle$$

provided these limits exist.

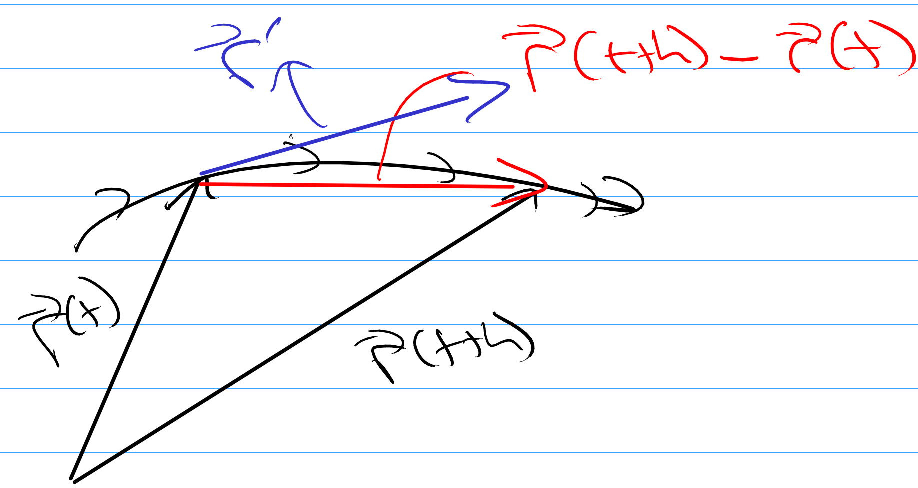
③ Continuity

$$\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$$

Derivative?

$$\frac{d}{dt}\{\vec{r}(t)\} = \lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

$$\vec{r}\,'(t) = \lim_{h\to 0} \frac{1}{h}\left(\vec{r}(t+h) - \vec{r}(t)\right)$$



$$\vec{r}'(t) = \lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

$$= \lim_{h\to 0} \frac{\langle f(t+h), g(t+h), h(t+h)\rangle - \langle f(t), g(t), h(t)\rangle}{h}$$

$$= \lim_{h\to 0} \left\langle \boxed{\frac{f(t+h) - f(t)}{h}}, \frac{g(t+h) - g(t)}{h}, \frac{h(t+h) - h(t)}{h} \right\rangle$$

$$\boxed{\vec{r}'(t) = \langle f', g', h' \rangle}$$

$$\int \vec{r}(t)\,dt = \left\langle \int f\,dt, \int g\,dt, \int h\,dt \right\rangle + \vec{C}$$

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b f\,dt, \int_a^b g\,dt, \int_a^b h\,dt \right\rangle$$

if $\int \vec{r}(t)\,dt = \vec{R}(t) + \vec{C}$

$$\int_a^b \vec{r}(t)\,dt = \vec{R}(b) - \vec{R}(a)$$