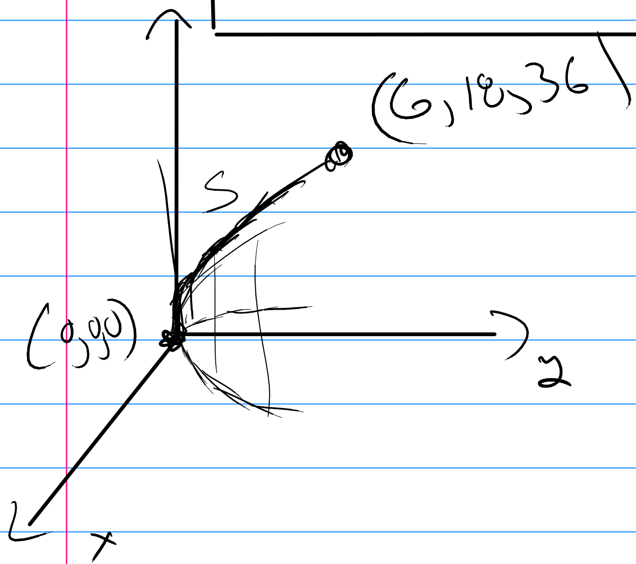
Math 344

Q's/ 10.8 #7 $x^2 = 2y$ $3z = xy$

in $\mathbb{R}^3$:

$$\boxed{L = \int_a^b |\vec{r}\,'|\,dt = \int_a^b \sqrt{(x')^2 + (y')^2 + (z')^2}\,dt \qquad \vec{r} = \langle x(t), y(t), z(t) \rangle}$$



$\frac{1}{2}x^2 = y$ $z = \frac{1}{3}xy$

$x = t$

$y = \frac{1}{2}t^2$

$z = \frac{1}{3}(t)(\frac{1}{2}t^2)$

$z = \frac{1}{6}t^3$

$\vec{r} = \langle t, \frac{1}{2}t^2, \frac{1}{6}t^3 \rangle$

$t \xrightarrow{?} (0,0,0)$ $a=0$

$t \xrightarrow{?} (6, 18, 36)$ $b=6$

$$L = \int_0^6 \sqrt{(1)^2 + (t)^2 + (\tfrac{1}{2}t^2)^2}\,dt$$

$$L = \int_0^6 \sqrt{1 + t^2 + \tfrac{1}{4}t^4}\,dt$$

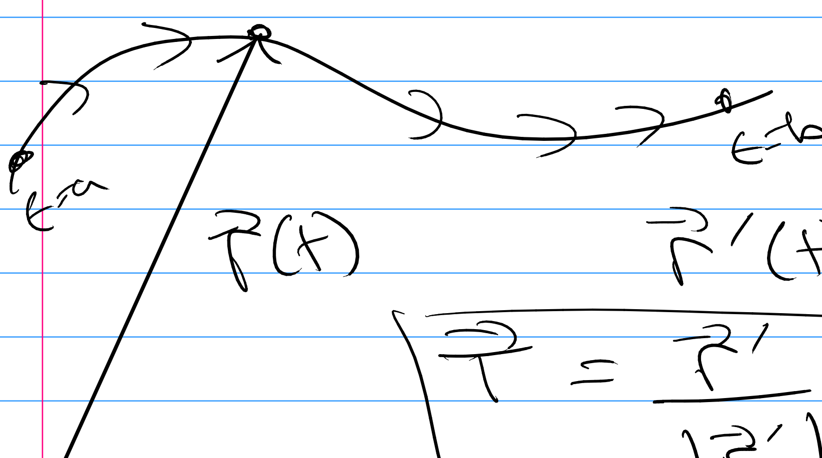
$$L = \frac{1}{2}\int_0^6 \sqrt{\underset{(2)^2}{4} + \underset{2\cdot(2)(t^2)}{4t^2} + \underset{(t^2)^2}{t^4}}\,dt$$

$$= \frac{1}{2}\int_0^6 \sqrt{(2+t^2)^2}\,dt$$

$$= \frac{1}{2}\int_0^6 |2+t^2|\,dt$$

Note: $\sqrt{\square^2} = |\square| = \begin{cases} \square, & \square \geq 0 \\ -\square, & \square < 0 \end{cases}$

$$\frac{1}{2}\int_0^6 (2+t^2)\,dt = \text{Finish}$$



$\vec{r}\,'(t)$ is tangent to $\vec{r}$

$$\boxed{\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|}}$$ unit tangent

$$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{|\vec{T}\,'|}{|\vec{r}\,'|} = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$$

$$s = s(t) = \int_a^t |\vec{r}\,'(u)|\,du$$

$$s = s(t) \xrightarrow[t]{\text{solve for}} t = t(s)$$

$$\vec{r}(t) = \vec{r}(t(s)) = \vec{r}(s)$$

$\boxed{\text{TNB frame}}$

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|}$$

$$\vec{N} = \frac{\vec{T}\,'}{|\vec{T}\,'|}$$

$$\vec{B} = \vec{T} \times \vec{N}$$

$\boxed{10.9}$ Motion in Space

$\boxed{\text{Before}}$ $s(x)$

$s'(x) = \text{Velocity}$

$s''(x) = v'(x) = \text{acceleration}$

$$s(x) \underset{\int dx}{\overset{\text{deriv}}{\rightleftarrows}} v(x) \underset{\int dx}{\overset{\text{deriv}}{\rightleftarrows}} a(x)$$

$$\int a(x)\,dx = v(x) + \text{Const}$$

$$\int v(x)\,dx = s(x) + \text{const}$$

$\mathbb{R}^3$

$\vec{r}(t)$ position

$\vec{r}'(t) = \vec{v}(t)$ velocity

$\vec{r}''(t) = \vec{v}'(t) = \vec{a}(t)$ acceleration

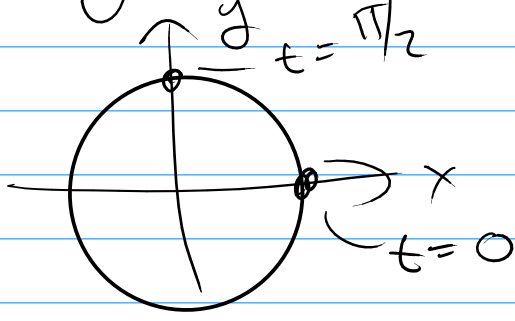
$$\int \vec{a}(t)\,dt = \vec{v}(t) + \vec{C}_1$$

$$\int \vec{v}(t)\,dt = \vec{r}(t) + \vec{C}_2$$

eqns of motion $\vec{r}, \vec{v}, \vec{a}$

Derivative Type Problem

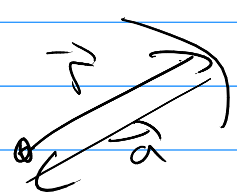
ex given $\vec{r}(t) = \langle \cos t, \sin t, 0 \rangle$



$\vec{v}(t) = \langle -\sin t, \cos t, 0 \rangle$

$\vec{a}(t) = \langle -\cos t, -\sin t, 0 \rangle$

Notice: $\vec{a}(t) = -\vec{r}$



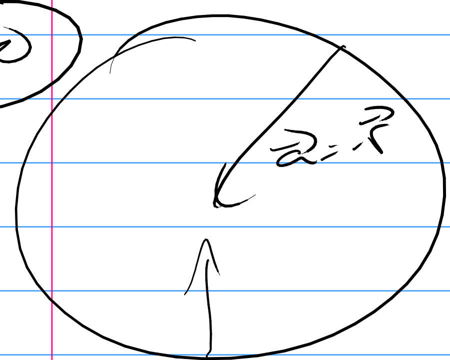
Now: Newton's 2nd Law

$$\text{Force} = \Delta\,(\text{Momentum})$$

change in (Mass)(Velocity)

$$\vec{F} = \frac{d}{dt}(m \cdot \vec{v})$$

$$\vec{F} = m \cdot \vec{v}\,' = m \cdot \vec{a}$$



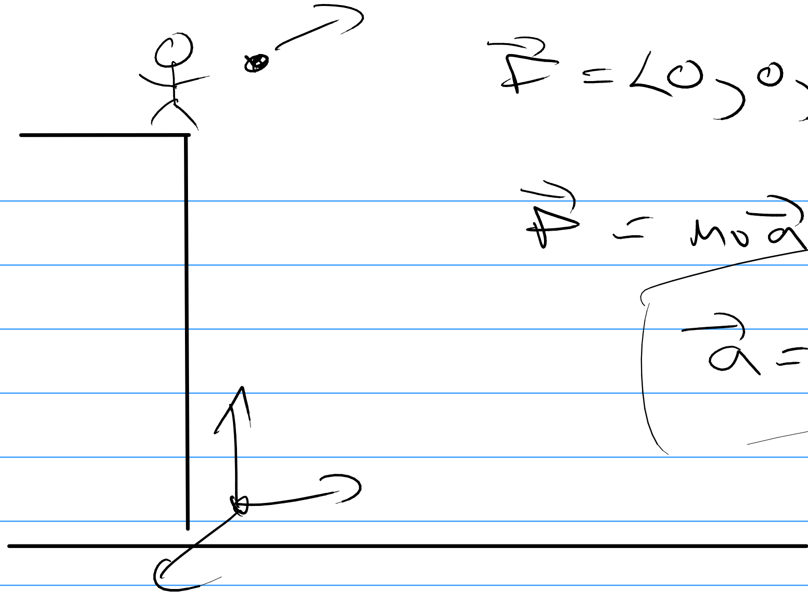
$$\vec{F} = m \cdot \vec{a}$$

$$\vec{F} = m(-\vec{r}) = -m\,\vec{r}$$

Integral Type: $\vec{a} \rightarrow \vec{v} \rightarrow \vec{r}$

Normally we can observe and measure accelerations / Forces..

$$\vec{F} = m \cdot \vec{a} \quad (\text{Known})$$



$\vec{F} = \langle 0, 0, -mg \rangle$

$\vec{F} = m\vec{a}$

$\boxed{\vec{a} = \langle 0, 0, -g \rangle}$

$\int \vec{a}\, dt$

$\int \vec{v}\, d(t)$