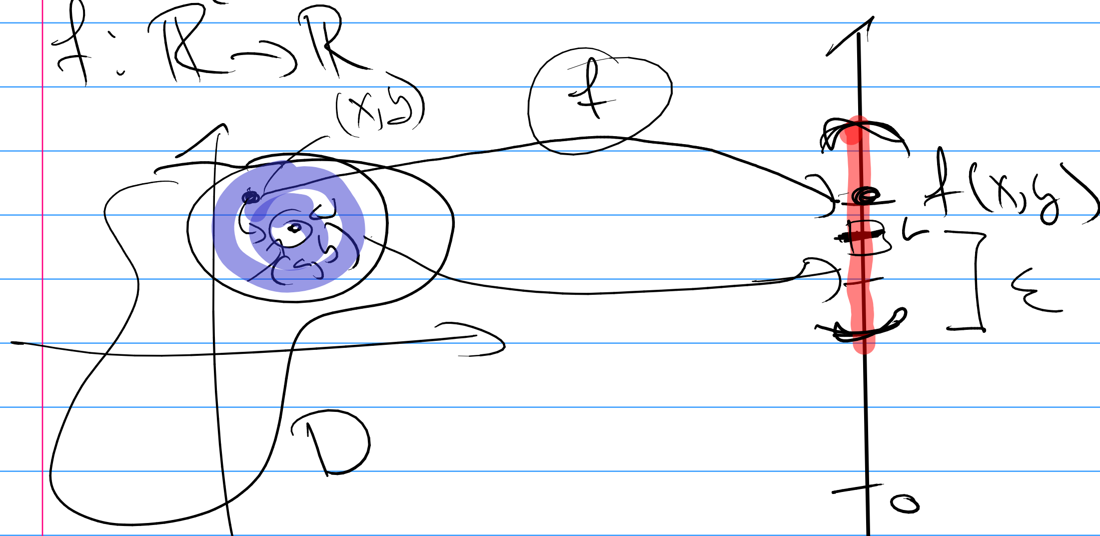
Math 344

11.2 Limits

$f: \mathbb{R}^2 \to \mathbb{R}$



$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

eqn of circle radius δ $\quad (x-a)^2 + (y-b)^2 = \delta^2$

<u>In it:</u> $\quad (x-a)^2 + (y-b)^2 \leq \delta^2$

In w/o boundry $\quad \sqrt{(x-a)^2 + (y-b)^2} < \delta$

Def: $(x,y) \in D \quad 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \Rightarrow |f(x,y) - L| < \varepsilon$

true then $\lim_{(x,y)\to(a,b)} f(x,y) = L$

$\boxed{\text{Proof}}$ show using a Mathematical argument that something is true.

Algebra: $\boxed{\text{Solve}}$

$$x^2 - x = 6$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \quad x = -2$$

$$\frac{x = 2}{(x)^2 = (2)^2} \qquad \text{Soln } 2$$
$$x^2 = 4$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2 \quad x = 2 \qquad \text{Soln } 2, \cancel{-2}$$

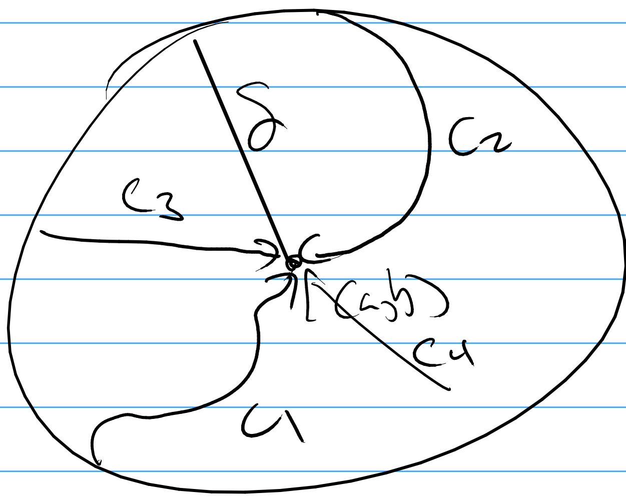
check

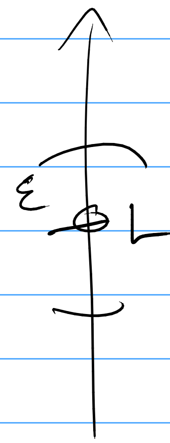
$$\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\lim_{(x,y)\to(a,b)} f(x,y) = L \quad \boxed{\underline{\text{show}}}$$

Means:

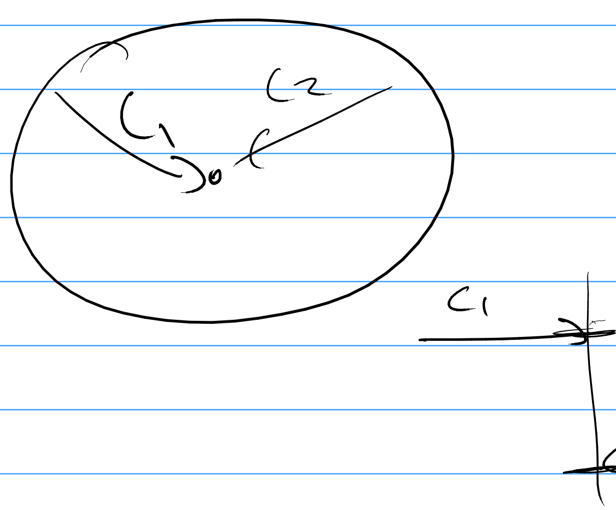
$$\underbrace{0 < \sqrt{(x-a)^2 + (y-b)^2} < \boxed{\delta}}_{\text{assume this}} \longrightarrow \underbrace{|f(x,y) - L| < \varepsilon}_{\text{show this}}$$





Two Probs for us

① Show $\lim_{(x,y)\to(a,b)} f(x,y)$ $\underline{\text{d.n.e.}}$



$$\lim_{(x,y)\xrightarrow{C_1}(a,b)} f = L_1$$

$$\neq$$

$$\lim_{(x,y)\xrightarrow{C_2}(a,b)} f = L_2$$

$\boxed{\text{d.n.e}}$

② guess @ L start backward reasoning

want: $\longrightarrow$ $|f(x,y) - L| < \varepsilon$

algebra

$$\sqrt{(x-a)^2 + (y-b)^2} < \text{stuff w/ } \varepsilon$$

$\parallel$

try δ

Now: go forward let $\delta =$ (found)

$$0 < \sqrt{(x-a)^2 + (y-b)^2} < (\quad)$$

algebra

$$|f(x,y) - L| < \varepsilon$$

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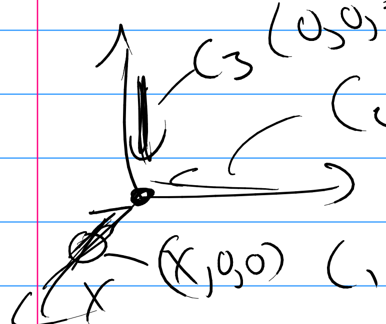
ex $$\lim_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y) \overset{?}{=} e^{1}\cos(0) = e$$

Show: $0 < \sqrt{(x-1)^2+(y+1)^2} < \delta \Rightarrow \left| e^{-xy}\cos(x+y) - e \right| < \varepsilon$

ex $$\lim_{(x,y)\to(0,0)} \frac{x^4-4y^2}{x^2+2y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2-2y^2)\cancel{(x^2+2y^2)}}{\cancel{(x^2+2y^2)}}$$

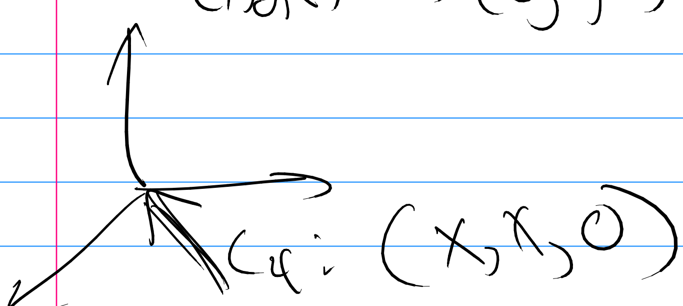
$$= \lim_{(x,y)\to(0,0)} x^2-2y^2 = 0$$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz^2+xz^2}{x^2+y^2+z^2}$$ dne



C_4 $x=y=z$

$$\lim_{(x,y,z)\xrightarrow{C_1}(0,0,0)} \frac{0}{x^2} = 0$$

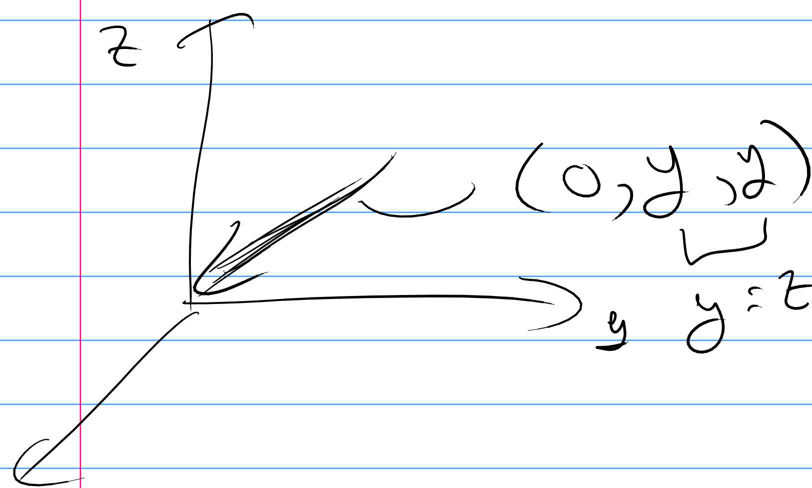


$$\lim_{\substack{(x,y,z)\to(0,0,0)\\ C_4}} f(x,y,z) = \lim \frac{x^2+0+0}{x^2+x^2+0}$$

$$= \lim \frac{x^2}{2x^2} = \boxed{\frac{1}{2}}$$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz^2+xz^2}{x^2+y^2+z^2} = \lim \frac{y^3}{2y^2} = \frac{1}{2}\lim y$$

$$= \boxed{0}$$



$$\boxed{\frac{1}{2} \neq 0}$$

d.n.e.

Limit Does Exist.

Continuous. $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

if Cont. for all $(a,b) \in D$

f is cont. on D