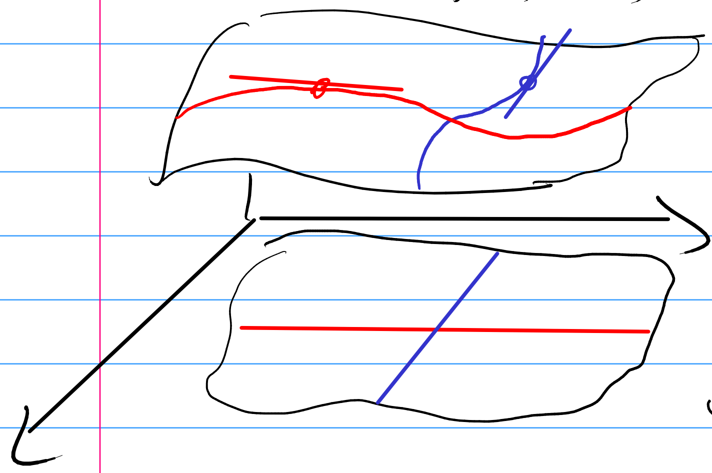
Math 344

11.3 Partial Derivatives

$f(x_1, x_2, x_3, \ldots, x_n) \to \mathbb{R}$



Partial derivative of $f(x_1, x_2, \ldots, x_n)$ with respect to x_i is the ordinary derivative of f with respect to x_i where all $x_j \neq x_i$ are constants.

$$\frac{d}{dx}\left[3 + 2x^2 + \sin(2x)\right]$$

$$= 0 + 4x + \cos(2x) \cdot 2$$

$$\frac{\partial}{\partial x}\left[3 + 2x^2 + \sin(2x)\right]$$

$$= 0 + 4x + \cos(2x) \cdot 2$$

$$\frac{\partial}{\partial x}\left[z + yx^2 + \sin(w \cdot x)\right]$$

$$= 0 + 2yx + \cos(w \cdot x) \cdot w$$

(ex) $f(x,y,z) = xy + y\ln(z) + z^{1/3}$

Find: f_x, f_y, f_z

$f_x = y + 0 + 0 = y$

$f_y = x + \ln(z) + 0 = x + \ln(z)$

$f_z = 0 + y\frac{1}{z} + \frac{1}{3}z^{-2/3} = \frac{y}{z} + \frac{1}{3z^{2/3}}$

Notation: $f_x(x,y,z) = \frac{\partial}{\partial x}\left[f(x,y,z)\right] = D_x\left[f(x,y,z)\right]$

1st order partials

$\boxed{f_x(x,y,z) = y}$

$f_y(x,y,z) = x + \ln z$

$f_z(x,y,z) = \frac{y}{z} + \frac{1}{3z^{2/3}} = \frac{1}{z}\left(y + \frac{1}{3}z^{1/3}\right)$

---

b/c these 1st order partials are functions $\Rightarrow$ we can take these new functions partial derivatives

1st order $\rightarrow$ 2nd order $\rightarrow$ 3rd order $\rightarrow$ etc ex $f(x,y)$

1st order   2nd order

$$f_x = \frac{\partial}{\partial x}[f] \qquad f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f\right) = \frac{\partial^2 f}{\partial x^2}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f\right)$$

$$f_y = \frac{\partial}{\partial y}[f] \qquad f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f\right)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} f\right) = \frac{\partial^2 f}{\partial y^2}$$

ex $f(x,y) = x^2 y^3 + \tan(x)\sin(y)$

1st order: $f_x = 2xy^3 + \sec^2(x)\sin(y)$

$f_y = 3x^2y^2 + \tan(x)\cos(y)$

2nd order: $f_{xx} = 2y^3 + 2(\sec x)^1(\sec x \tan x)\sin y$

$= 2y^3 + 2\sec^2 x \tan x \sin y$

$f_{xy} = 6xy^2 + \sec^2(x)\cos(y)$

$f_{yx} = 6xy^2 + \sec^2(x)\cos(y)$

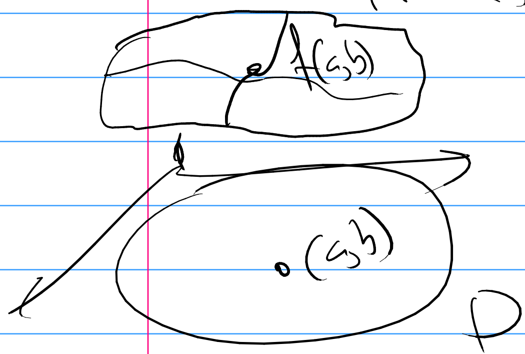
same!

$f_{yy} = 6x^2y - \tan(x)\sin(y)$

Noticed on above $f_{xy} = f_{yx}$

$\boxed{\text{Clairaut's Th}^{\text{m}}}$ $f(x,y)$

if f is defined on a disk D



contains (a,b) if

f_{xy} and f_{yx} are continuous on disk D

then $f_{xy} = f_{yx}$

Ordinary Differential Eqn.

$$y'' + y = 0$$

$$\frac{d^2y}{dx^2} + y = 0$$

$\boxed{\text{Solve:}}$ → means find $y = f(x)$ that makes eqn true.

Guess: $y'' + y = 0$ A soln is $\boxed{y = \sin(x)}$

$y'' + y = 0$ guess $y = \sin(x)$

check:

$y = \sin(x)$

$y' = \cos(x)$

$y'' = -\sin(x)$

$y'' + y =$

$-\sin x + \sin x = 0$

Partial Differential Eqn

eqn with partial derivatives.

Laplace's Eqn $\quad u_{xx} + u_{yy} = 0 \qquad u = f(x,y)$

All we will do is verify you have a Soln.

ex is $f(x,y) = \sin(xy)$ a Soln to Laplace's Eqn?

$f_x = y\cos(xy) \qquad f_y = x\cos(xy)$

$f_{xx} = -y^2\sin(xy) \qquad f_{yy} = -x^2\sin(xy)$

$f_{xx} + f_{yy} = (-y^2\sin(xy)) + (-x^2\sin(xy))$

$\neq 0$ No.