Math 344

Q's / (10.8) $y = 9e^x = f(x)$

Max Curvature.

$$\kappa = \left|\frac{dT}{ds}\right| = \frac{|T'|}{|r'|} = \frac{|r' \times r''|}{|r'|^3}$$

$y = f(x)$

$$\boxed{\kappa(x) = \frac{|f''|}{\left(1+(f')^2\right)^{3/2}}}$$

$f = 9e^x \qquad f' = 9e^x \qquad f'' = 9e^x$

Calc 3

$$\boxed{\kappa(x) = \frac{9e^x}{\left(1+81e^{2x}\right)^{3/2}}}$$

Calc 1

Maximize

$$\kappa'(x) = \frac{9e^x\left(1+81e^{2x}\right)^{\cancel{3/2}\,1} - 9e^x \frac{3}{2}\cancel{\left(1+81e^{2x}\right)}^{1/2}\left(162e^{2x}\right)}{\left(1+81e^{2x}\right)^{\cancel{6/2}\,5/2}}$$

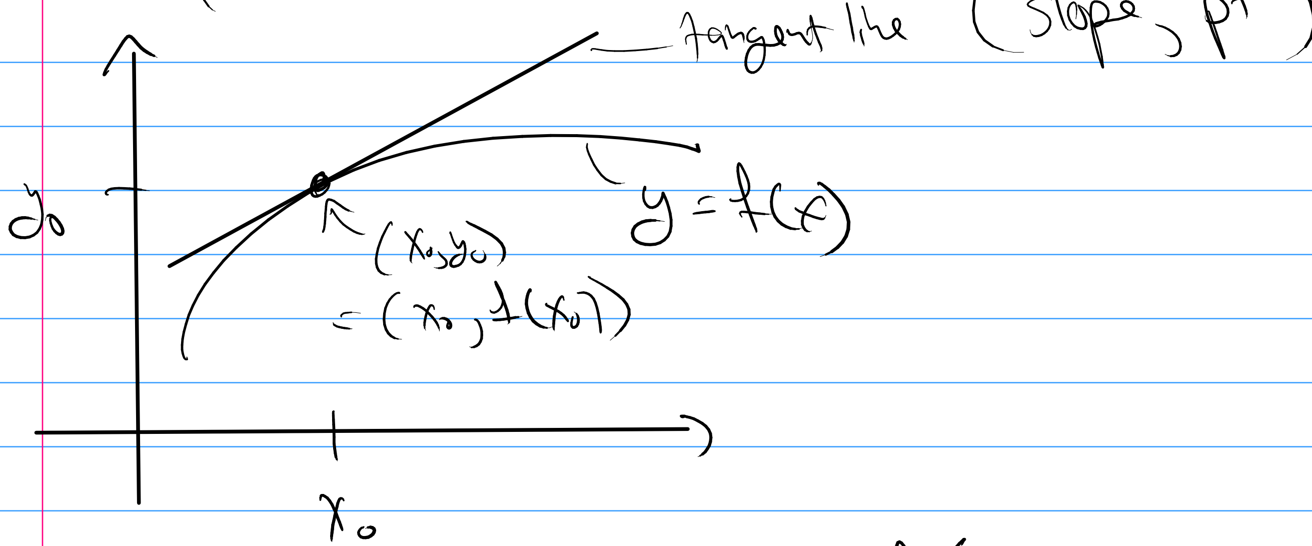
$$= \frac{9e^x\left[1+81e^{2x} - 3\cdot 81e^{2x}\right]}{\left(1+81e^{2x}\right)^{5/2}}$$

$$K' = \frac{(9e^x)\left[1 - 162e^{2x}\right]}{(1+81e^{2x})^{5/2}}$$

possible max @ $1 - 162e^{2x} = 0$

Finish

11.4 Linear Approximations



tangent line: $y - y_0 = f'(x_0)(x - x_0)$

$$y = y_0 + f'(x_0)(x - x_0)$$

our tangent line

$$y = f(x_0) + f'(x_0)(\boxed{x} - x_0)$$

So near (x_0, y_0)

$$\boxed{f(x) \approx f(x_0) + f'(x_0)(x - x_0)}$$

Local linear approximation.

$\sqrt{4.1} = ?$ $\qquad f(x) = \sqrt{x} \quad f'(x) = \frac{1}{2\sqrt{x}}$

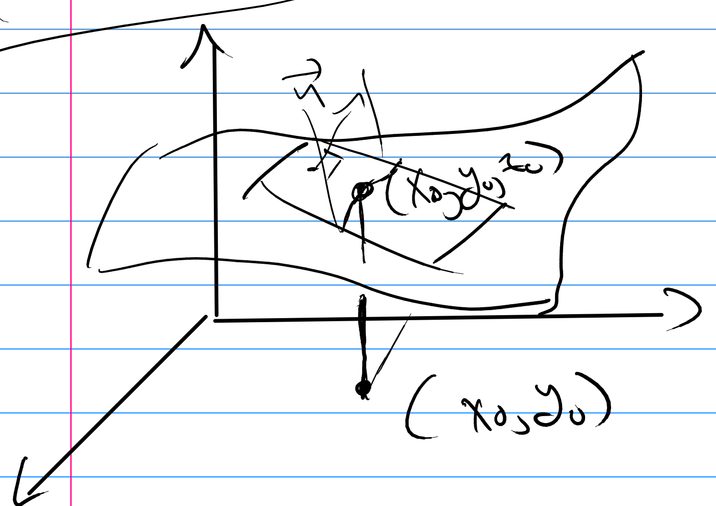
$$\sqrt{4.1} \approx \sqrt{4} + \frac{1}{2\sqrt{4}}(4.1 - 4)$$

$$\sqrt{4.1} \approx 2 + \frac{1}{4}(.1) = \boxed{2.025}$$

$\boxed{z = f(x,y)}$ ① ; $\quad w = f(x,y,z)$ $\qquad$ Linear Approx.

tangent plane: (normal, pt)

$$\vec{n} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$



$z_0 = f(x_0, y_0)$

$$\langle a, b, c\rangle \cdot \langle x - x_0, y - y_0, z - z_0\rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

solve for z. $= p(x,y) \leftarrow$ plane's function

$$z = z_0 + \underbrace{\frac{a}{c}}_{=m_1}(x - x_0) + \boxed{\underbrace{\frac{b}{c}}_{=m_2}(y - y_0)}$$

$y = y_0$ trace.

$x = x_0$ trace

$$m_2 = f_y(x_0, y_0)$$

So plane's function:

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$$

So near (x_0, y_0)

$$f(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$$

Similar: $w = f(x,y,z)$

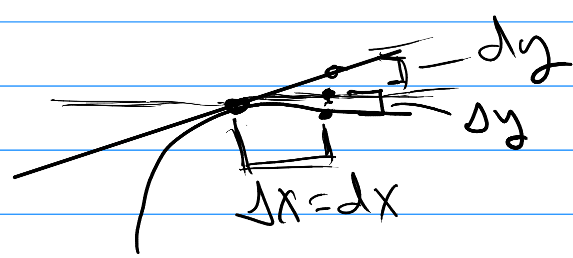
$$\boxed{\begin{aligned} f(x,y,z) \approx f(x_0,y_0,z_0) &+ f_x(x_0,y_0,z_0)(x-x_0) \\ &+ f_y(x_0,y_0,z_0)(y-y_0) \\ &+ f_z(x_0,y_0,z_0)(z-z_0) \end{aligned}}$$

Local Liner Approx near (x_0,y_0,z_0)

Application: real error $\Delta z \approx$?

Tools: Differentiable $\to$ linear approx is good

Thm: Partial derivs exist and are cont. near a point $\to$ f is differentiable at the point.



$$dy \approx \Delta y$$