Math 344

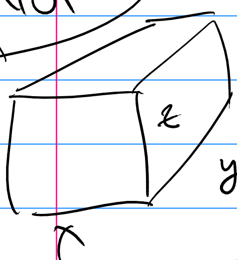
11.4 Error Propagation.. $F(x_1, x_2, \dots, x_n)$

$$\Delta F \approx dF = F_{x_1}\underbrace{dx_1}_{\text{error in } x_1} + F_{x_2}dx_2 + \dots + F_{x_n}dx_n$$

example:

Vol

$$V = x \cdot y \cdot z$$



$$\Delta V \approx dV = V_x\,dx + V_y\,dy + V_z\,dz$$

$$\Delta V \approx yz\,dx + xz\,dy + xy\,dz$$

ex $x = 15\text{ft}$ $|dx| = .1/12$ $y = 30\text{ft}$ $|dy| = .1/12$ $z = 5\text{ft}$ $|dz| = .1/12$

$$\Delta V \approx (30)(5)\left(\frac{.1}{12}\right) + (15)(5)\left(\frac{.1}{12}\right) + (15)(30)\left(\frac{.1}{12}\right)$$

$$\approx \frac{15}{12} + \frac{7.5}{12} + \frac{45}{12} = \frac{67.5}{12} = 5.625$$

$$\Delta V \approx \pm 5.625 \text{ ft}^3$$

11.5

calc 1 $\left[f(g(x)) \right]'$ ord. deriv of composition

ex $\frac{d}{dx}\left[\sin\left(\sqrt{x^3+1}\right)\right]$

⇒ Chain rule:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

ex $\frac{d}{dx}\left[\sin\left(\sqrt{x^3+1}\right)\right]$

$$= \cos\left(\sqrt{x^3+1}\right) \cdot \left(\frac{1}{2\sqrt{x^3+1}}\right) \cdot (3x^2)$$

What if ...

$f(x_1, x_2, .., x_n)$ and $x_i(t_1, t_2, -, t_m)$

So $f\Big(x_1(t_1, t_2, .., t_m), x_2(t_1, t_2, .., t_m), \ldots\Big)$

ex $\boxed{f(x,y,z) = x + yz}$ $\quad x(s,t) = s+t$

$y(s,t) = st$

$z(s,t) = s^2$

$$\to f(s+t, st, s^2) = \underbrace{(\overset{x}{s+t}) + (\overset{y}{st})(\overset{z}{s^2})}$$

f is really a function of s, t

So $\underline{\underline{f_s, f_t}}$ are what we want.

Chain Rule Ver #1 $\quad f(x,y)$ and $x(t), y(t)$

$\to f$ is really a function of only t.

$$\boxed{\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}}$$

ex $f(x,y) = x^3 + xy$ & $x(t) = \sin(t)$

$y(t) = t^3 + 1$

$$\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} = (3x^2 + y)\cos t + x(3t^2)$$

Case #2

$$\boxed{\begin{gathered} f(x,y) \quad \text{and} \quad x(s,t),\ y(s,t) \\ \frac{\partial f}{\partial s} = f_x \frac{\partial x}{\partial s} + f_y \frac{\partial y}{\partial s} \\ \frac{\partial f}{\partial t} = f_x \frac{\partial x}{\partial t} + f_y \frac{\partial y}{\partial t} \end{gathered}}$$

In General $f(x_1, x_2, \dots, x_n)$

and $x_1(t_1, t_2, \dots, t_m)$

$x_2(t_1, t_2, \dots, t_m)$

$\vdots$

$x_n(t_1, t_2, \dots, t_m)$

f is really a function of $(t_1, t_2, \dots, t_m)$

$$\left[\frac{\partial f}{\partial t_1} = f_{x_1} \frac{\partial x_1}{\partial t_1} + f_{x_2} \frac{\partial x_2}{\partial t_1} + f_{x_3} \frac{\partial x_3}{\partial t_1} + \dots + f_{x_n} \frac{\partial x_n}{\partial t_1} \right.$$

$$\left[\frac{\partial f}{\partial t_2} = f_{x_1} \frac{\partial x_1}{\partial t_2} + \dots + f_{x_n} \frac{\partial x_n}{\partial t_2} \right.$$

$\vdots$ etc $\qquad \vdots$ etc

$$f(x,y,z,w) = xy + z^2 w$$

$$x(s,t) = s + t^2$$
$$y(s,t) = st$$
$$z(s,t) = s^3$$
$$w(s,t) = t^2$$

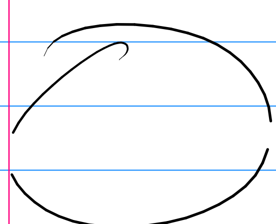
$$\frac{\partial f}{\partial s} = f_x \frac{\partial x}{\partial s} + f_y \frac{\partial y}{\partial s} + f_z \frac{\partial z}{\partial s} + f_w \frac{\partial w}{\partial s}$$
$$= (y)(1) + (x)(t) + (2zw)(3s^2) + (z^2)(0)$$

$$\frac{\partial f}{\partial t} = (\text{@ home for fun})$$

Application

Implicit Differentiation.

calc1 Find $\frac{d}{dx}[y]$ when: ex $\boxed{y^3 x + \sin(y) = xy}$

ex  $\boxed{x^2 + y^2 = 1}$ ← Implicit $\boxed{y = \;\;}$

explicit expression

$y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$ ← explicit of only x's

Idea is to create $F(x,y)=0$ from

Implicit equation..

ex $y^3 x + \sin(y) = xy$

$$\boxed{y^3 x + \sin(y) - xy} = 0$$

$$\overset{\shortparallel}{F(x,y)}$$ b/c $y = y(x)$

we have one variable

$$F_x \frac{dx}{dx} + F_y \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{F_x}{F_y}}$$

ex $\rightarrow$ $$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$