Math 349

$\boxed{\text{Exam 1}}$

1) $\vec{r} = \langle t^3+1,\ t+1,\ \sin t \rangle$ tangent line.

$\vec{r}\,' = \langle 3t^2,\ 1,\ \cos t \rangle = \langle 0, 1, 0 \rangle$

Solve

$3t^2 = 0 \quad \cos t = 0$

$t = 0 \neq t = \pi/2$ Never

$\int (2t^2 + t)\,dt$ Power rules.

$\int t \sin(t^2)\,dt$ Substitution

$\int \ln t\,dt$ Parts!

2) $l = \boxed{\int_a^b |\vec{r}\,'|\,dt}$

$$l = \int_0^1 \sqrt{(0)^2 + (t)^2 + (t^2)^2}\,dt$$

$$= \int_0^1 \sqrt{t^2 + t^4}\,dt$$

$$= \int_0^1 |t| \sqrt{1+t^2}\,dt$$

$$= \int_0^1 t\sqrt{1+t^2}\,dt$$ Use Substitution

③ $\vec{r} = \int \vec{a}\,dt = (\quad) + \vec{c}$

$\vec{v} = \langle 2t, 3t^2, 4t^3 \rangle + \vec{c}$

$\vec{v}(0) = \vec{0} + \vec{c} = \langle 1, 0, 0 \rangle$

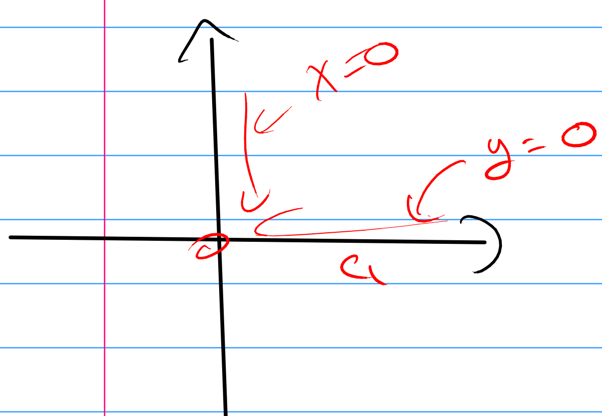
$\vec{v} = \langle 2t+1, 3t^2, 4t^3 \rangle$

Same idea for

$\vec{r} = \int \vec{v}\,dt = (\quad) + \vec{c}$

etc

④ $\lim\limits_{(x,y)\to(0,0)} \dfrac{2y^4 \cos^3 x}{x^4 + y^4}$? 0



$c_1\ (y=0)$

$\lim\limits_{x\to 0} \dfrac{0}{x^4} = \lim\limits_{x\to 0} 0 = 0$

$c_2\ (x=0)$

$\lim\limits_{y\to 0} \dfrac{2\cancel{y^4}}{\cancel{y^4}} = \lim\limits_{y\to 0} 2 = 2$

$\nexists$ dne

5 $f = x\sin(y^2+z) + y$

$f_x = \sin(y^2+z)$

$f_y = 2xy\cos(y^2+z) + 1$

$f_z = x\cos(y^2+z)$

$H = yx^2 + y^2x^3 + x\sin z$

$H_x = 2yx + 3y^2x^2 + \sin z$

$H_{xx} = 2y + 6y^2x$

$H_{xxy} = 2 + 12yx$

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6 $V = \pi r^2 h$

$\Delta V \approx dV = V_r\,dr + V_h\,dh$

$= (2\pi rh)dr + (\pi r^2)dh$

2 10 .1 etc

$= \pm 4.4\pi$

(7) $\underline{\underline{\text{use:}}}$ $w_s = w_x x_s + w_y y_s + w_z z_s$

$\underline{\underline{\text{etc}}}$

(8) $D_{\vec{u}} f = \nabla f \cdot \vec{u}$ ← $\underline{\underline{\text{unit}}}$

$\boxed{\langle f_x, f_y \rangle}$ @ $(0, \pi/2) \to \langle 1, 0 \rangle$

$\vec{u} = \dfrac{\langle -6, 8 \rangle}{10}$

$D_{\vec{u}} f = \boxed{-3/5}$

(9) $f = x \cdot e^{-2x^2 - 2y^2}$

$f_x = (1 + x(-4x))\, e^{-2x^2-2y^2} = \underline{\underline{(1-4x^2)}} e^{\sim}$

$f_y = \underline{\underline{-4xy}}\, e^{-2x^2-2y^2}$

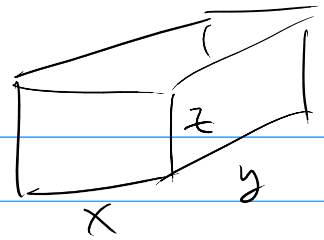
$\underline{\underline{\text{critical:}}}$ $f_x = 0$ $\quad (1-4x^2) = 0 \to \boxed{x = \pm 1/2}$

$f_y = 0 \quad x \cdot y = 0 \to y = 0$

$(1/2, 0) \quad (-1/2, 0)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

(10)



$$f = V = \boxed{xyz}$$

$$g = \boxed{xy + 2xz + 2yz} = 12$$

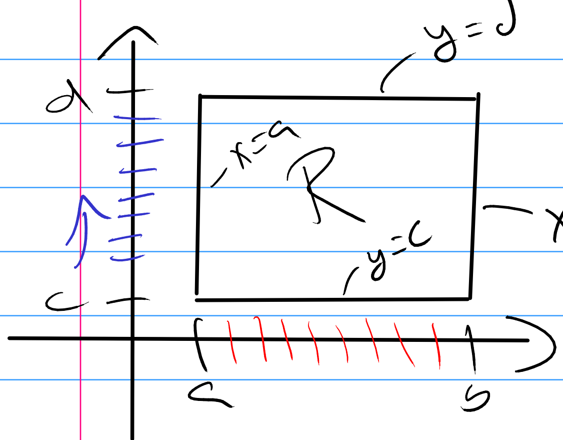
$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g = 12 \end{cases}$$

finish

$$V = \iint_R f(x,y)\, dA$$

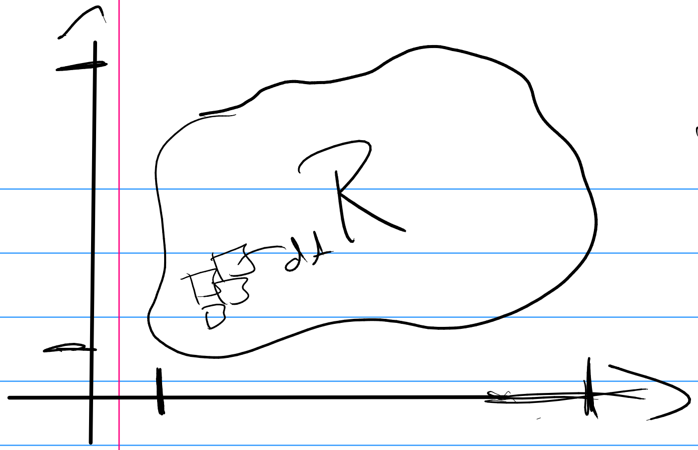
<u>Double Integrals.</u>

$$= \lim_{\max \Delta x, \Delta y \to 0} \sum_j \sum_i f(x_{ij}^*, y_{ij}^*)\, \Delta A_{ij}$$



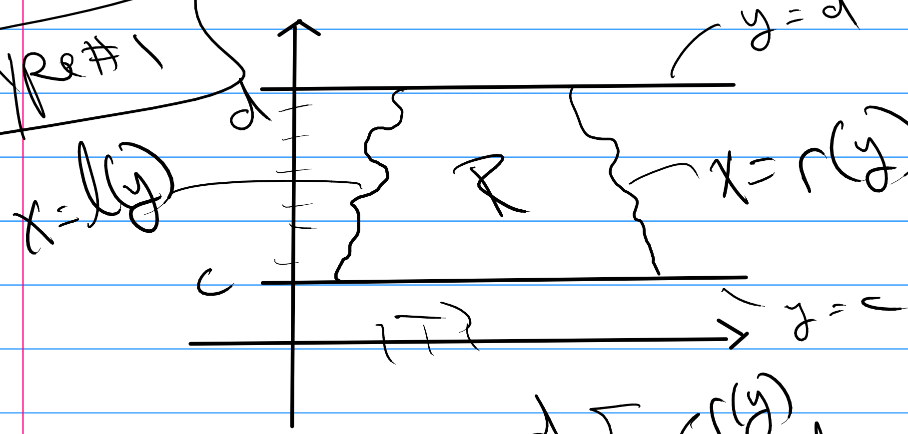
$$V = \int_c^d \left(\int_a^b f(x,y)\, dx \right) dy$$

$$= \int_a^b \left[\int_c^d f\, dy \right] dx$$



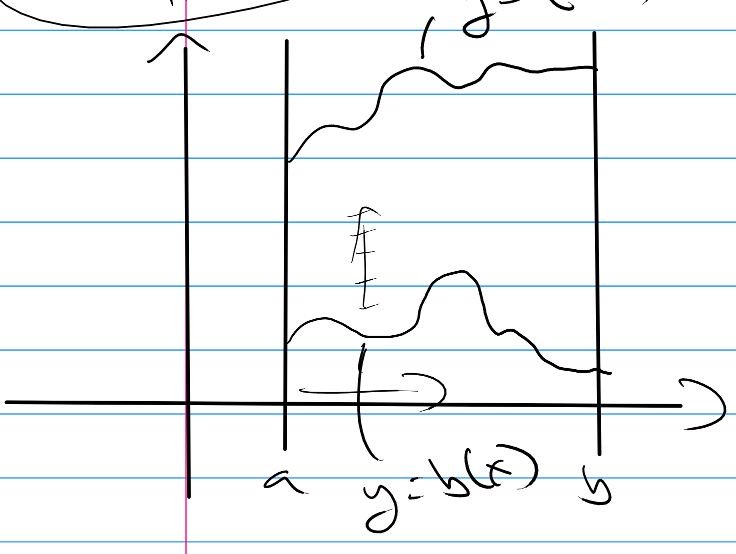
$$V = \iint_R f \, dA$$

Type #1

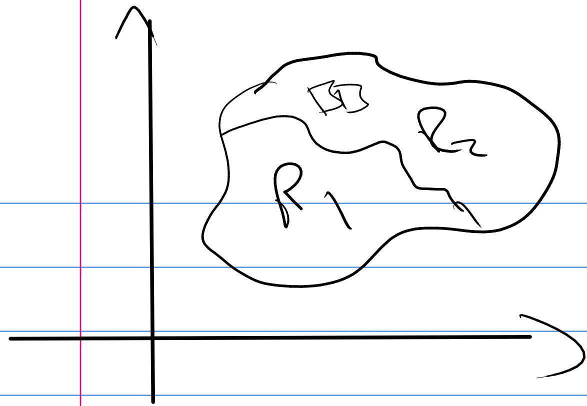


$$V = \int_c^d \left[\int_{l(y)}^{r(y)} f \, dx \right] dy$$

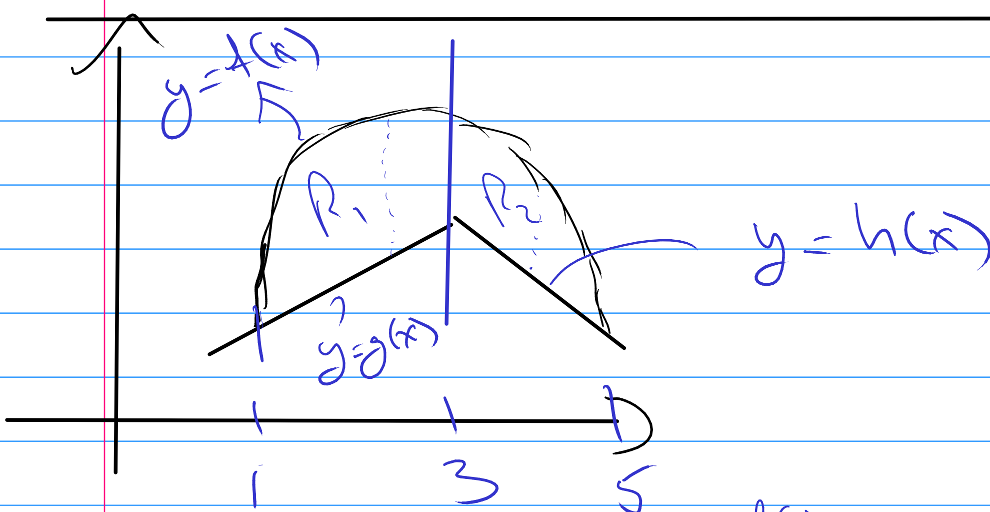
Type #2



$$\int_a^b \left[\int_{b(x)}^{t(x)} f \, dy \right] dx$$



$$\iint_R f\,dA = \iint_{R_1} f\,dA + \iint_{R_2} f\,dA$$



$$R_1 = \int_1^3 \left[\int_{g(x)}^{f(x)} F\,dy \right] dx + \int_3^5 \left[\int_{h(x)}^{f(x)} F\,dy \right] dx$$