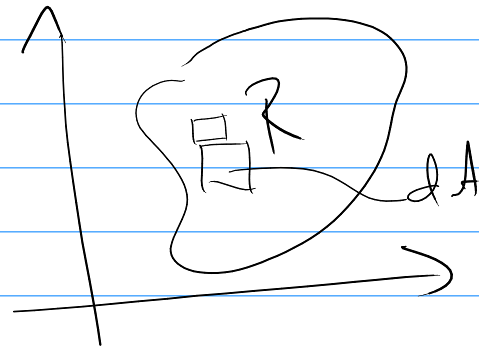
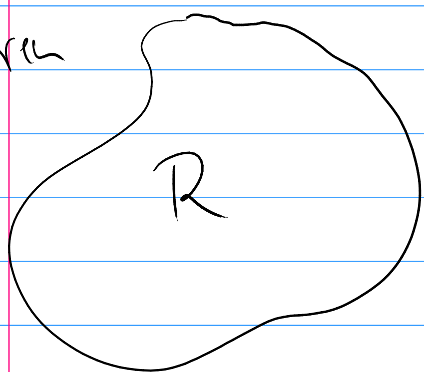
Math 344

$$\iint_R \underbrace{f}_{\text{height}} \; \underbrace{dA}_{\text{area}}$$

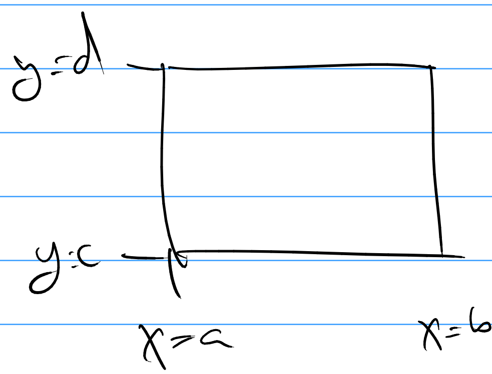


Given

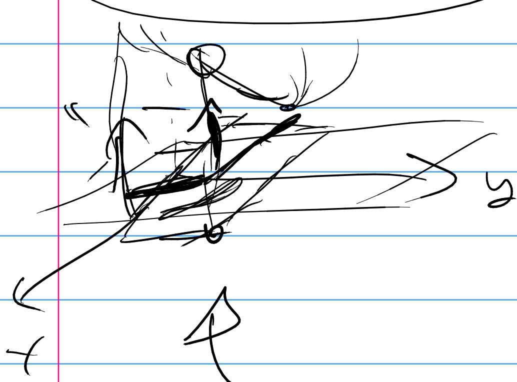


ex x's $-$ y's

$$\underbrace{[a,b]}_{x\text{-interval}} \times \underbrace{[c,d]}_{y\text{-interval}}$$



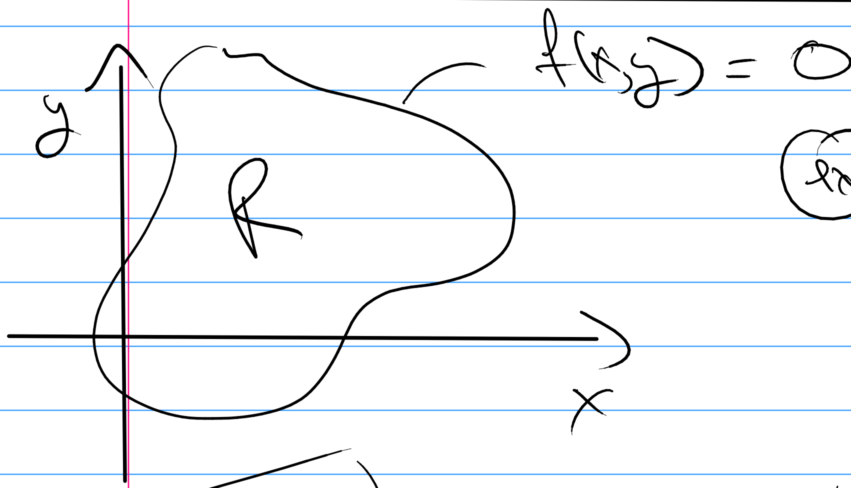
ex z = 4 + x^2 + (y − 2)^2 and the planes z = 1, x = −2, x = 2, y = 0, and y = 2



$$\iint_R 4 + x^2 + (y-2)^2 \, dA$$

$$- \iint_R 1 \, dA$$

$$= \int_{-2}^{2}\left(\int_{0}^{2}\left[3 + x^2 + (y-2)^2\right] dy\right) dx$$



$\boxed{\text{Cartesian coord.}}$

ex $\boxed{x^2+y^2-4=0}$

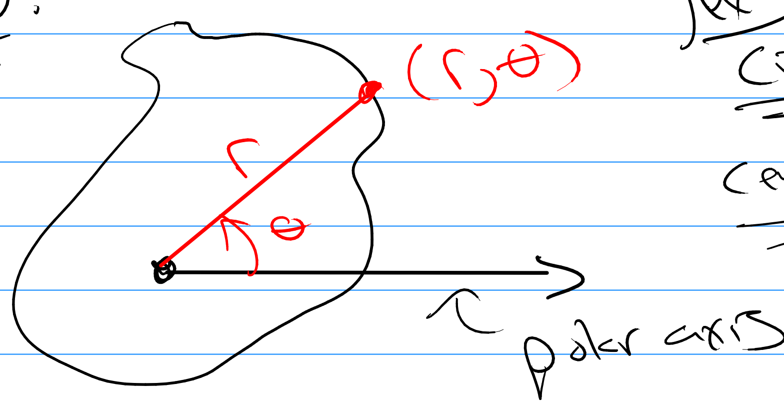
Implicit

explicit

$$y = +\sqrt{4-x^2}$$

$$y = -\sqrt{4-x^2}$$

Polar Coord:



ex

circle centered @ origin.

$\boxed{r=2}$

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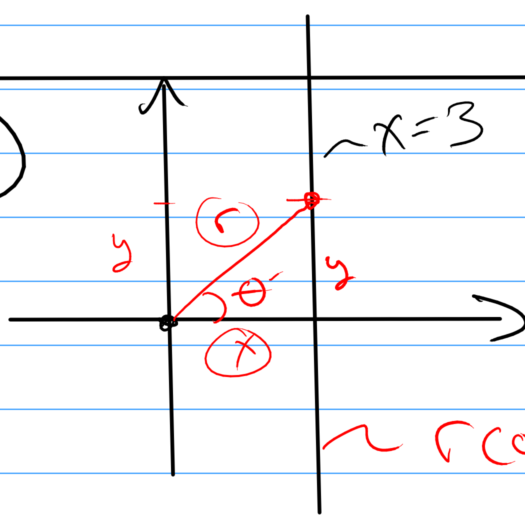
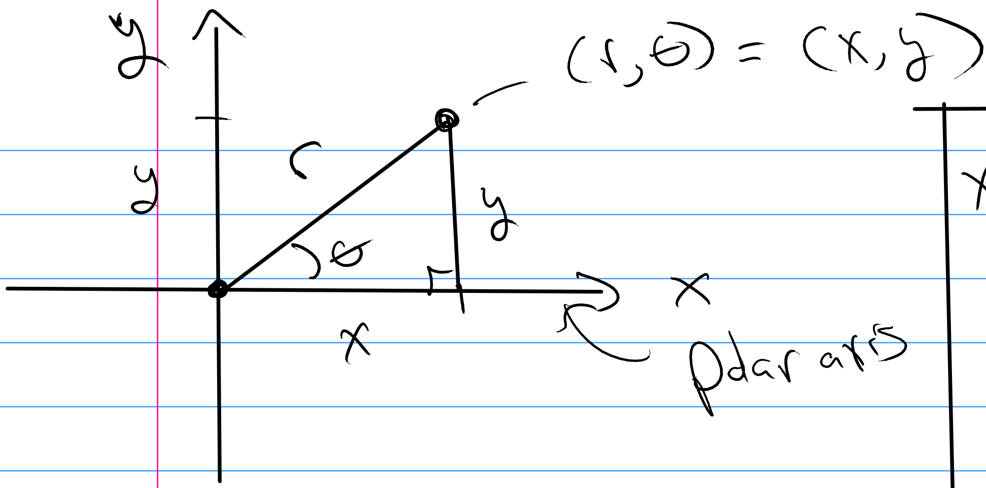
ex



$$\cos\theta = \frac{x}{r}$$

$$x = r\cos\theta$$

$$r\cos\theta = 3 \rightarrow r = \frac{3}{\cos\theta} = 3\sec\theta$$



$$\boxed{\begin{aligned} x^2 + y^2 &= r^2 \\ x &= r\cos\theta \\ y &= r\sin\theta \\ \frac{y}{x} &= \tan\theta \\ \frac{x}{y} &= \cot\theta \end{aligned}}$$

$\iint_R f \, dA$ using polar coord.

$A = \frac{1}{2}(\phi) r^2$

$$\rightarrow \quad dA = \frac{1}{2}(\Delta\theta) r_2^2 - \frac{1}{2}(\Delta\theta) r_1^2$$

$$dA = \frac{1}{2}(r_2^2 - r_1^2)\Delta\theta = \underbrace{\frac{1}{2}(r_2+r_1)}_{r_i^*}\overbrace{(r_2 - r_1)}^{\Delta r}\Delta\theta$$

$$dA = r\,dr\,d\theta$$

So $\iint_R f\,dA = \iint_R f\; r\,dr\,d\theta$

can only have r's and θ's

Ex) Polar Rectangle

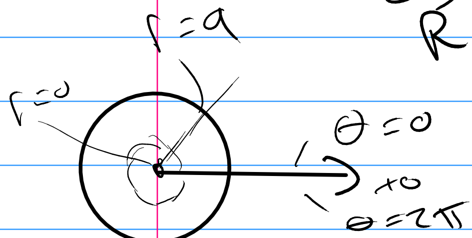
$$\iint_R f(x,y)\,dA$$

$$\int_{\theta_1}^{\theta_2}\left(\int_{r_1}^{r_2} f(r\cos\theta, r\sin\theta)\,r\,dr\right)d\theta$$

θ_1, r_1 constants.

Ex) Area of a circle of radius a.

$$\iint_R (1)\,dA = \text{area of } R$$



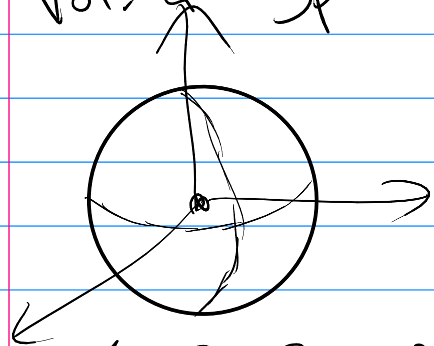
$$\int_0^{2\pi}\left(\int_0^a (1)\,r\,dr\right)d\theta$$

$$= \int_0^{2\pi}\left(\frac{1}{2}r^2\Big|_0^a\right)d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} a^2 \, d\theta = \frac{1}{2} a^2 \theta \Big|_0^{2\pi}$$

$$= \boxed{\pi a^2}$$

Ex Vol. of Sphere of radius a



$$V = \iint_R f \, dA$$

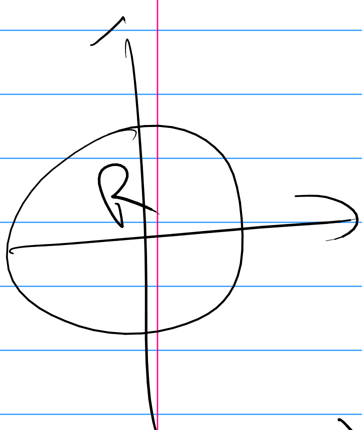
$$V = 2 \iint_R f \, dA$$

upper half of Shere

$x^2 + y^2 + z^2 = a^2$

upper: $z = \sqrt{a^2 - (x^2 + y^2)}$

$$V = 2 \iint_R \sqrt{a^2 - (x^2 + y^2)} \, dA$$



$$V = 2 \int_0^{2\pi} \int_0^a \sqrt{a^2 - \underbrace{(x^2+y^2)}_{r^2}} \; r \, dr \, d\theta$$

$$V = 2 \int_0^{2\pi} \left(\int_0^a r \sqrt{a^2 - r^2} \, dr \right) d\theta$$

$$= \text{finish} = \frac{4}{3} \pi a^3$$