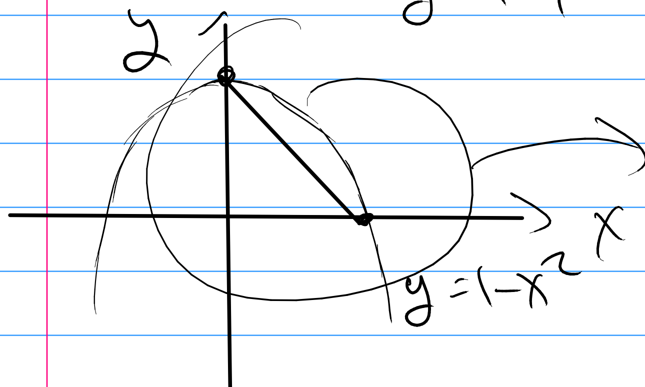
Math 344

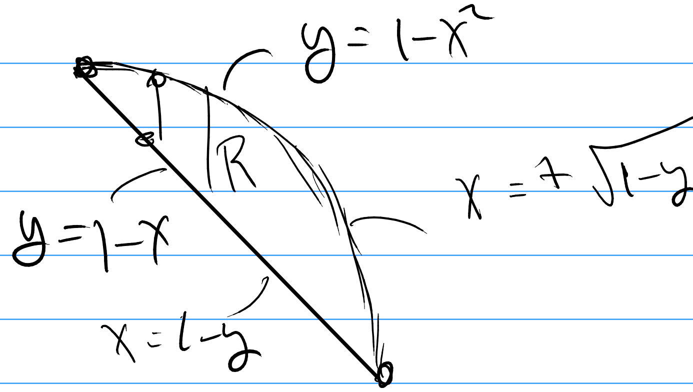
Q1s

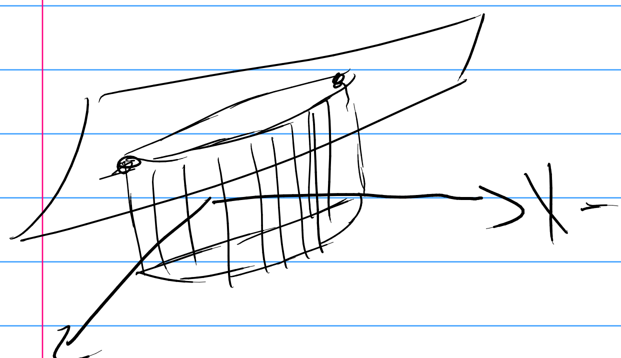
$x - 2y + z = 8 \to z = \boxed{8 - x + 2y}$

above $x + y = 1$ $\quad x^2 + y = 1 \quad 0 = 8 - x + 2y$

$y = 1 - x \quad y = 1 - x^2$







$$V = \iint_R (8 - x + 2y)\, dA$$

$$V = \int_0^1 \left(\int_{1-x}^{1-x^2} (8 - x + 2y)\, dy \right) dx$$

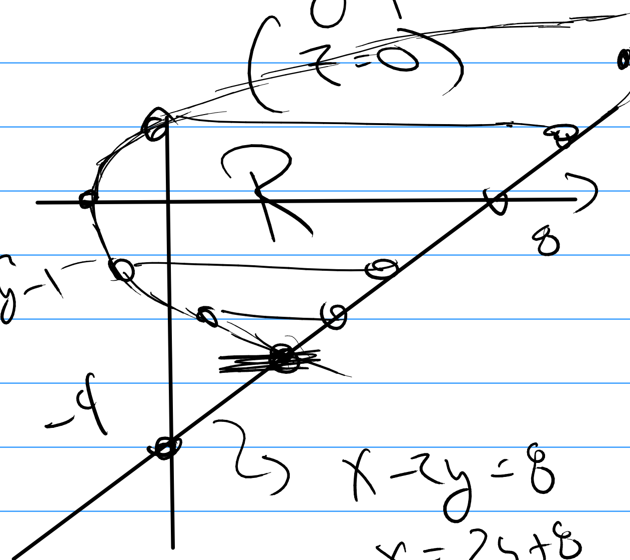
vs

$$V = \int_0^1 \left(\int_{1-y}^{\sqrt{1-y}} (8 - x + 2y)\, dx \right) dy$$

Similar Problem: find Vol. above x-y plane

for $x-2y+z=8$ and <u>above</u> $x=y^2-1$

(1) where does $x-2y+z=8$ cross xy plane



$$\rightarrow \quad x-2y+0=8$$

$$y=\tfrac{1}{2}x-4$$

add

$$x=\boxed{y^2-1}$$

$$\iint_R (8-x+2y)\,dA$$

$$\int_{bot}^{top}\left(\int_{y^2-1}^{2y+8}(8-x+2y)\,dx\right)dy$$

Intersections of $\begin{cases} x-2y=8 \Leftarrow x=(2y+8) \\ x=y^2-1 \end{cases}$

$$(2y+8)=y^2-1$$

$$y^2-2y-9=0$$

$$y=\frac{2\pm\sqrt{4+36}}{2}=\frac{2\pm\sqrt{40}}{2}$$

$y = 1 \pm \sqrt{10}$ bottom $1-\sqrt{10}$

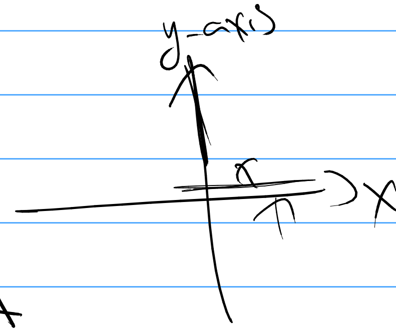
top $1+\sqrt{10}$

$$V = \int_{1-\sqrt{10}}^{1+\sqrt{10}} \left(\int_{y^2-1}^{2y+8} (8-x+2y)\,dx \right) dy$$

Apps: ① Vol. $\iint_R f\,dA$

② Area $\iint_R (1)\,dA$

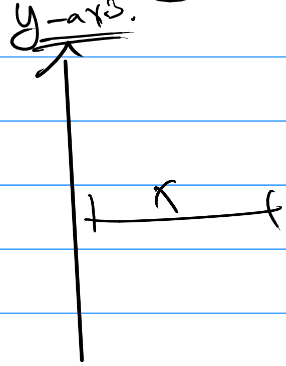
lamina ③ $M = \iint_R \rho\,dA$



$$\bar{x} = \frac{1}{M}\iint_R x\rho\,dA$$

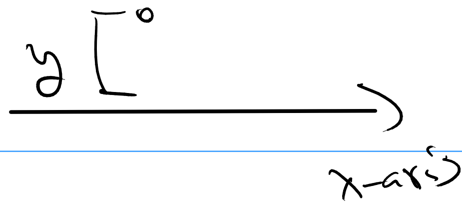
$$\bar{y} = \frac{1}{M}\iint_R y\rho\,dA$$

Rotational ④ $M = \iint_{e} \rho\,dA$

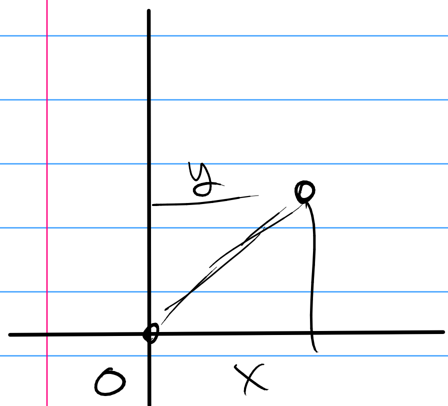


$$I_{y\text{-axis}} = \iint_R x^2 \rho\,dA$$

Moment of Inertia



$$I_{x\text{-axis}} = \iint_R y^2 \rho \, dA$$



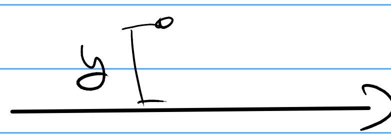
$$I_{origin} = \iint_R (x^2+y^2)\rho \, dA$$

$$= I_{y\text{-axis}} + I_{x\text{-axis}}$$

polar moment of Inertia.

$$mR^2 = I$$

x-axis:



$$m\bar{y}^2 = I_{x\text{-axis}} = \iint_R y^2 \rho \, dA$$

Apps: ① $\iint_R 1 \, dA$ ② $\iint (1) \, dA$

③ $\iint_R \rho \, dA$ ④ $\iint_R x\rho \, dA$

$\iint_R y\rho \, dA$

③ $\iint_R x^2 \rho \, dA$ $\qquad \iint_R (x^2+y^2)\rho \, dA$

$\iint_R y^2 \rho \, dA$

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ex ① density is inversely proportional to distance from origin

② boundary of lamina is $y = \sqrt{1-x^2}$

$y = \sqrt{4-x^2}$

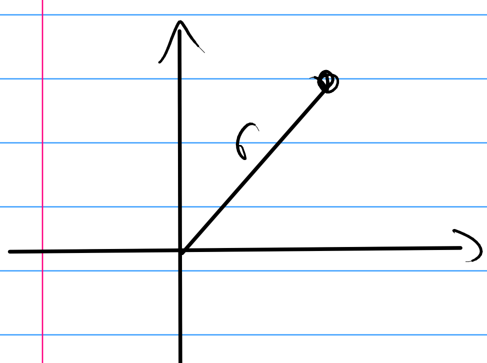
with x-axis to join them.

Mass?

$$\iint_R \rho \, dA$$

__Note:__ Directly Proportional $a \propto b \rightarrow a = k \cdot b$

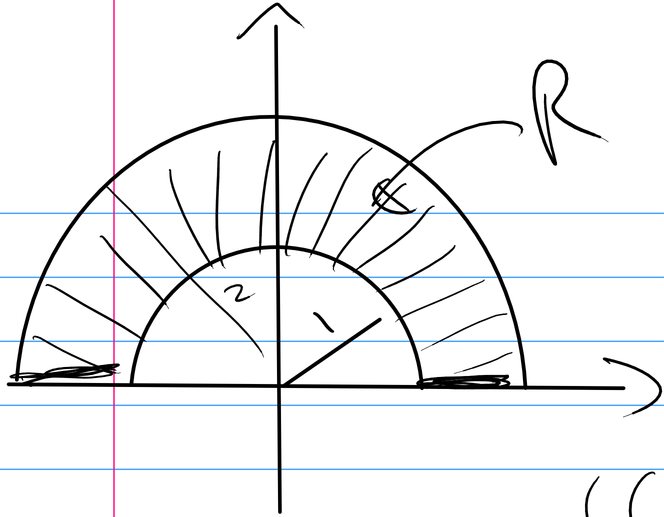
Inversely Proportional $a \propto \frac{1}{b} \rightarrow a = k \cdot \frac{1}{b}$



$\rho \propto \frac{1}{r}$ $\qquad \rho = k \cdot \frac{1}{r}$

$\rho(r,\theta) = \frac{k}{r}$

$\rho(x,y) = \frac{k}{\sqrt{x^2+y^2}}$



$$y = \sqrt{1-x^2}$$

$$y = \sqrt{4-x^2}$$

$$M = \iint_R \rho \, dA$$

$\boxed{\text{polar}}$

$$M = \int_0^{\pi} \int_1^2 \frac{k}{\cancel{r}} \cancel{r} \, dr \, d\theta$$

$$M = k \int_0^{\pi} \int_1^2 dr \, d\theta$$
