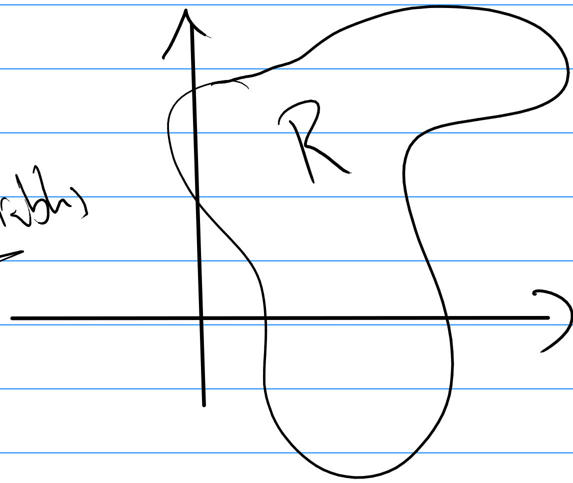
Math 344

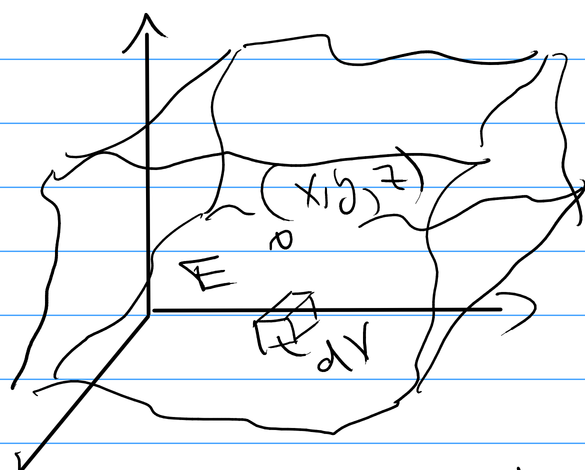
$\boxed{12.5}$ $f(x,y,z)$ vs $f(x,y)$

$f\underbrace{(x,y)}_{\text{Ind. Variables}}$



$$\iint_R f(x,y)\,dV$$

$f\underbrace{(x,y,z)}_{\text{Ind. Variables}}$

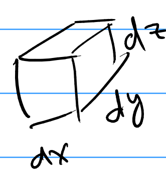


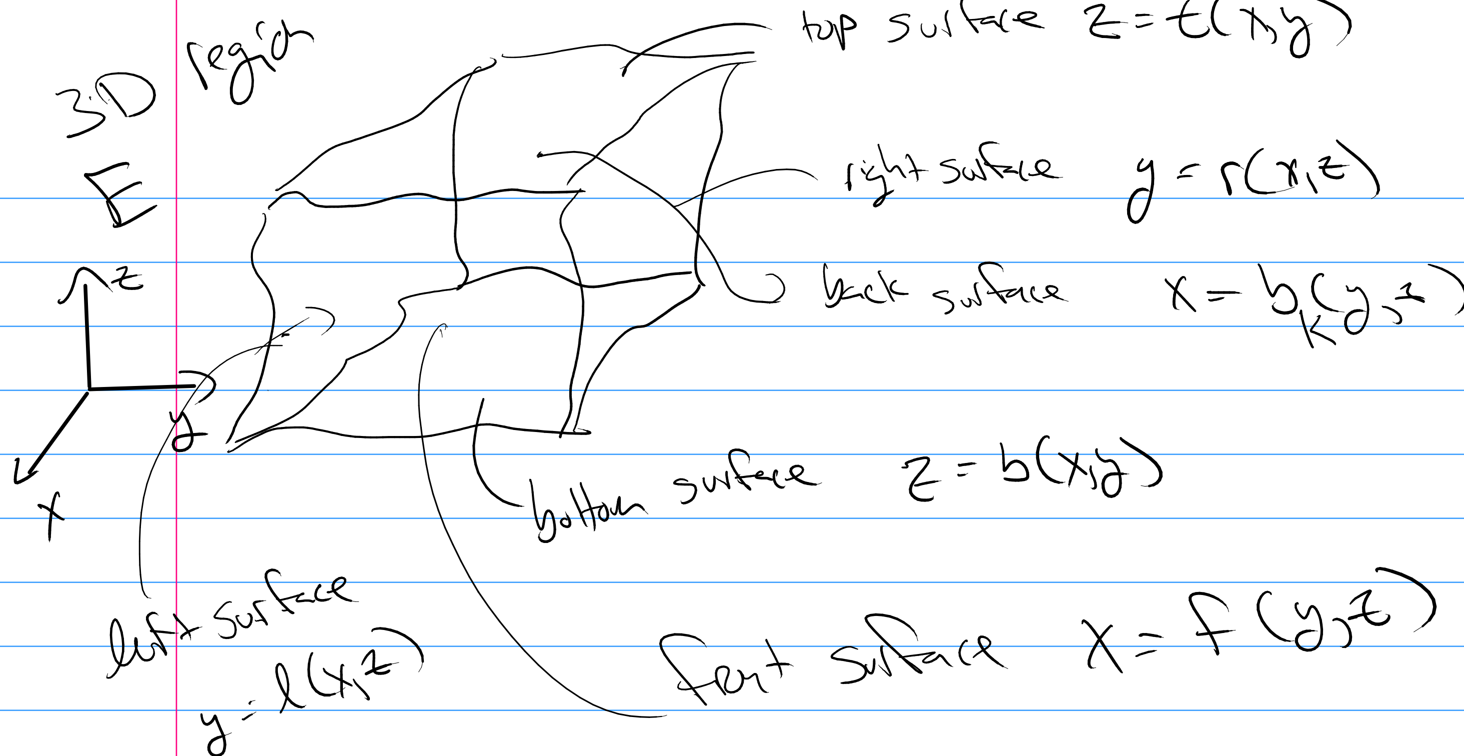
$$\lim \sum_k \sum_j \sum_i f(x_i^*, y_j^*, z_k^*)\,dV$$

$$\boxed{\iiint_E f(x,y,z)\,dV}$$

Cartesian coord.

$dV = dx\,dy\,dz$



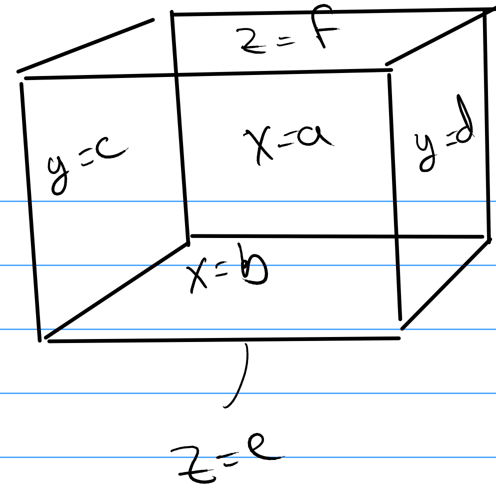


$$\iiint_E f\,dV$$

$$= \int_{\text{left}}^{\text{right}}\left(\int_{\text{bottom}}^{\text{top}}\left(\int_{\text{back}}^{\text{front}} f\,dx\right)dz\right)dy$$

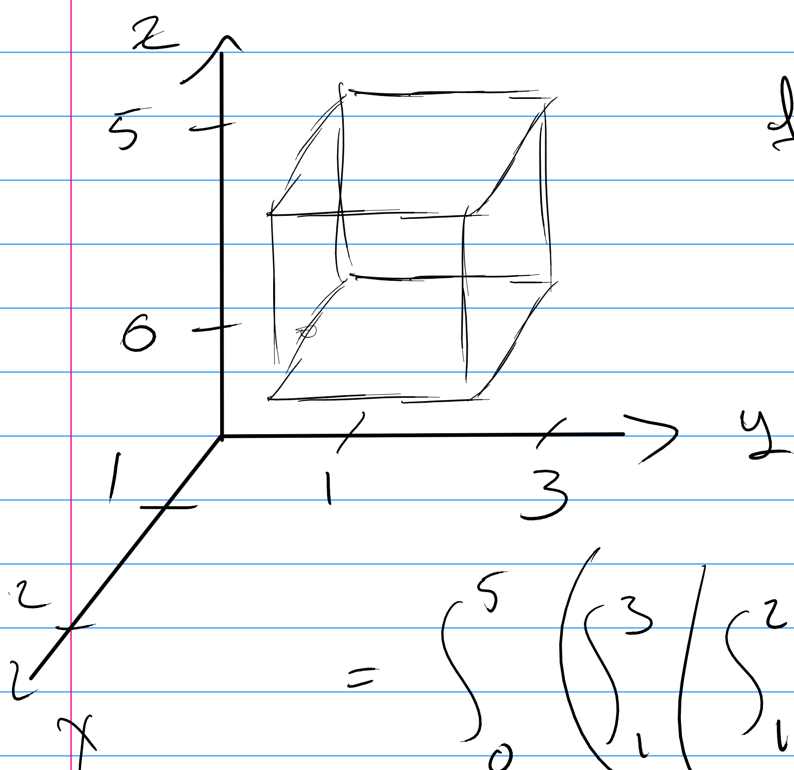
Note: you can re-arrange dx, dy, dz as needed. Just keep correct surface with them.

"Easy" → boxes



$a \le x \le b$, $c \le y \le d$, $e \le z \le f$

so $\iiint_E f\,dV = \int_a^b \left(\int_c^d \left(\int_e^f f\,dz \right) dy \right) dx$



$f(x,y,z) = x+y+z$

$\iiint_E (x+y+z)\,dV$

$$= \int_0^5 \left(\int_1^3 \left(\int_1^2 (x+y+z)\,dx \right) dy \right) dz$$

$$= \int_0^5 \int_1^3 \left[\underbrace{\frac{1}{2}x^2 + yx + zx \Big|_{x=1}^{x=2}}_{(2+2y+2z)-(\frac{1}{2}+y+z)} \right] dy\,dz$$

$$= \int_0^5 \left(\int_1^3 \left(3/2 + y + z \right) dy \right) dz$$

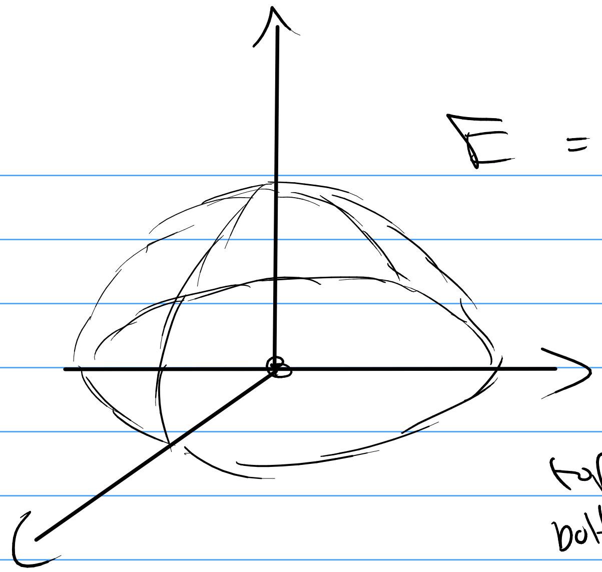
$$= \int_0^5 \left(\underbrace{\frac{3}{2}y + \frac{1}{2}y^2 + yz \Big|_{y=1}^{y=3}}_{\left\{\frac{9}{2} + \frac{9}{2} + 3z\right\} - \left[\frac{3}{2} + \frac{1}{2} + z\right]} \right) dz$$

$$= \int_0^5 (7 + 2z)\, dz = \underline{\underline{\text{Finish!}}}$$

What if we don't have a box?

$$\iiint_E f \, dV$$

__Note:__ $\displaystyle\int_{z=e}^{z=f} \left(\int_{y=\text{left}(z)}^{y=\text{right}(z)} \left(\int_{x=\text{back}(y,z)}^{x=\text{front}(y,z)} f \, dx \right) dy \right) dz$



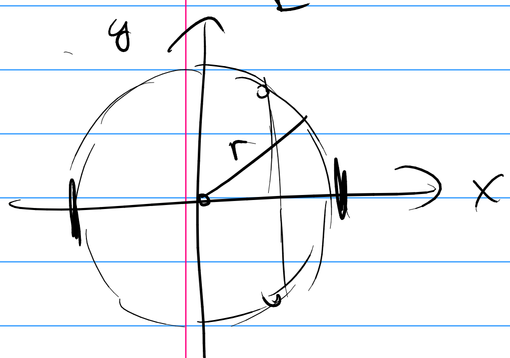
E = upper 1/2 of sphere

$$x^2+y^2+z^2=r^2$$

top $\rightarrow$ $z=\sqrt{r^2-x^2-y^2}$

bottom $\rightarrow$ $z=0$

$$\iiint_E f\,dV = \int_{x=-r}^{x=r}\left(\int_{y=-\sqrt{r^2-x^2}}^{y=\sqrt{r^2-x^2}}\left(\int_{z=0}^{z=\sqrt{r^2-x^2-y^2}} \boxed{f}\,dz\right)dy\right)dx$$



ex

$$\int_0^{\sqrt{\pi}}\left(\int_0^{x}\left(\int_{y=0}^{y=xz} x^2\sin y\,dy\right)dz\right)dx$$

$$\left(-x^2\cos y\Big|_{y=0}^{y=xz}\right)$$

$$-x^2\cos(xz)+x^2$$

$$=\int_0^{\sqrt{\pi}}\left(\int_{z=0}^{z=x}\left(x^2-\underbrace{x^2\cos(xz)}_{u=xz\quad \frac{du}{x}=dz}\right)dz\right)dx$$

$$= \int_0^{\sqrt{\pi}} \left[x^2 z - x\sin(xz) \Big|_{z=0}^{z=x} \right] dx$$

$$= \int_0^{\sqrt{\pi}} \left[\left(x^3 - x\sin(x^2) \right) - (0) \right] dx$$

$$= \int_0^{\sqrt{\pi}} \left(\underbrace{x^3} - \underbrace{x\sin(x^2)} \right) dx$$

$$= \frac{1}{4}x^4 + \frac{1}{2}\cos(x^2) \Big|_0^{\sqrt{\pi}}$$

$$= \left(\frac{1}{4}\pi^2 + \frac{1}{2}\cos(\pi) \right) - \left(0 + \frac{1}{2} \right)$$

$$= \boxed{\frac{1}{4}\pi^2 - 1}$$