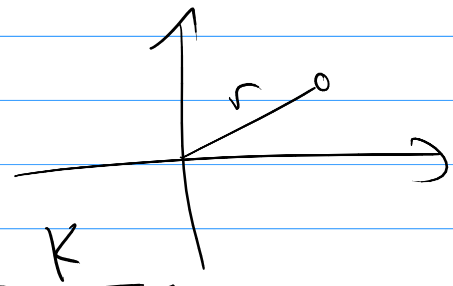
Math 344

~~Q5~~ 12.4 in → $x^2+y^2=18y$ out → $x^2+y^2=81$

$$\rho \propto \frac{1}{r}$$

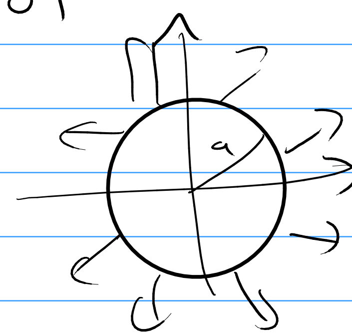


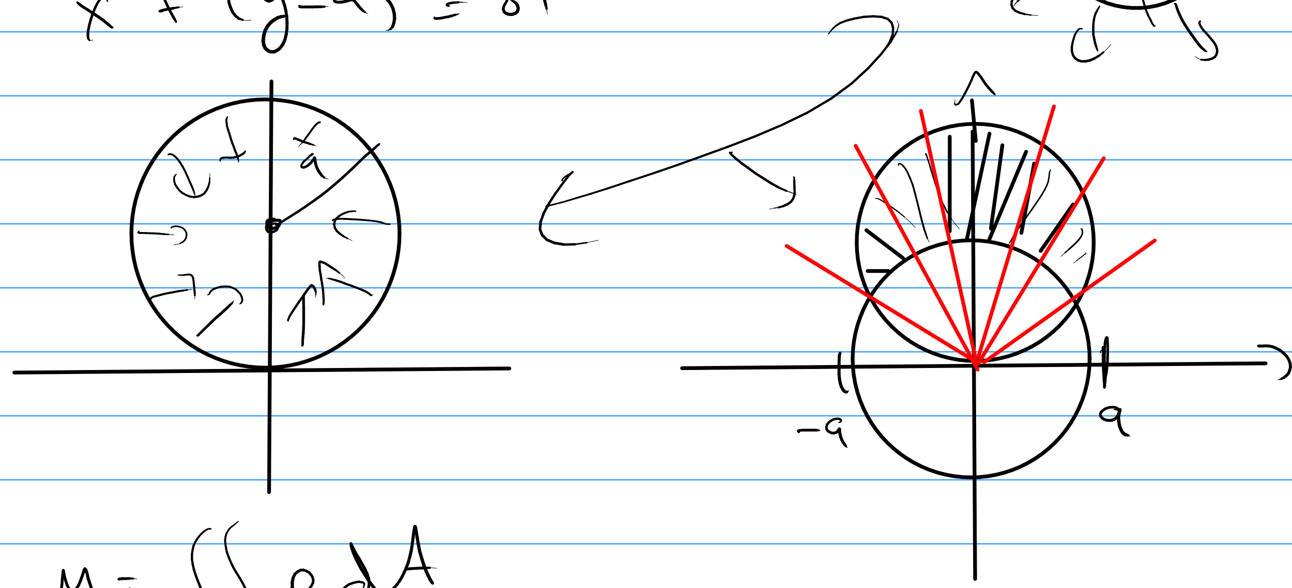
$$\rho(x,y) = \frac{K}{r} = \frac{K}{\sqrt{x^2+y^2}}$$

R in → $x^2+y^2=18y$ out → $x^2+y^2=81$

$$x^2+(y^2-18y+81) = 0+81$$

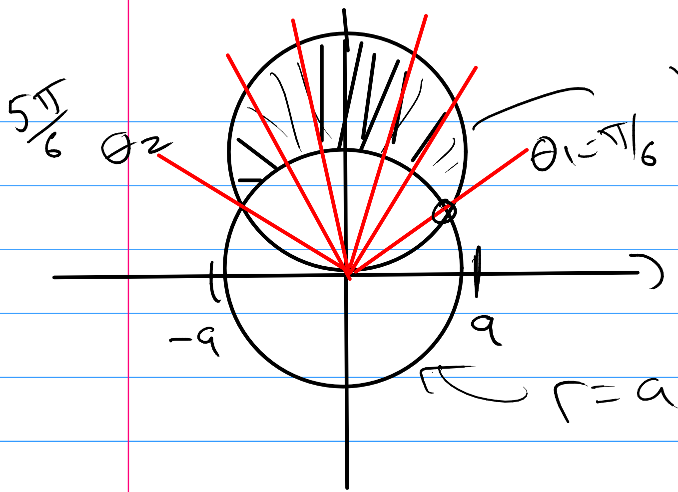
$$x^2+(y-9)^2=81$$





$$M = \iint_R \rho \, dA$$

$$M = \int_{?_0}^{?_0} \int_{?_0}^{?_0} \frac{K}{\cancel{r}} \cancel{r} \, dr \, d\theta$$



$x^2+y^2=18y$

$r=18\sin\theta$

Solve intersection

$9=18\sin\theta$

$\frac{1}{2}=\sin\theta$

$$M=K\int_{\pi/6}^{5\pi/6}\left(\int_{9}^{18\sin\theta}dr\right)d\theta$$

$$\bar{x}=\frac{1}{M}\iint_R x\rho\, dA$$

$$\bar{x}=\frac{K}{M}\int_{\pi/6}^{5\pi/6}\int_{9}^{18\sin\theta} r\cos\theta\, dr\, d\theta$$

$$\bar{y}=\frac{K}{M}\int_{\pi/6}^{5\pi/6}\int_{9}^{18\sin\theta} r\sin\theta\, dr\, d\theta$$

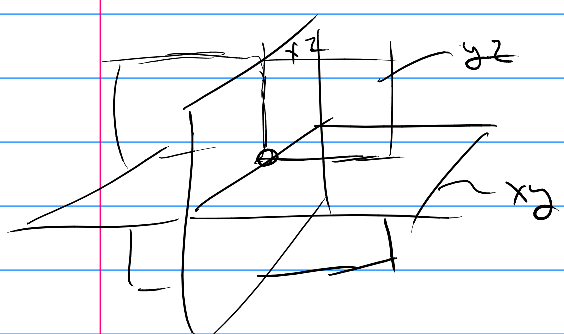
Apps $\iiint_E f(x,y,z)\,dV$

① $\iiint_E (1)\,dV = \text{Vol}(E)$

② $\rho(x,y,z)$ density in 3space

$$\left(\rho = \frac{\text{mass}}{\text{Vol}}\right)(\text{Vol}) = \text{mass}$$

$$m = \iiint_E \rho(x,y,z)\,dV$$



$$M_{yz\text{ plane}} = \iiint_E x\rho\,dV$$

$$\bar{x} = \frac{1}{m}\underbrace{\iiint_E x\rho\,dV}_{M_{yz\text{ plane}}}$$

$$\bar{y} = \frac{1}{m}\underbrace{\iiint_E y\rho\,dV}_{M_{xz\text{ plane}}}$$

$$\bar{z} = \frac{1}{m}\underbrace{\iiint_E z\rho\,dV}_{M_{xy\text{ plane}}}$$

③ $\sigma(x,y,z)$ $\frac{\text{charge}}{\text{Vol.}}$

$$\iiint_E \sigma \, dV$$

④ $$I_x = \iiint_E (y^2+z^2)\,\rho\, dV$$

$$I_y = \iiint_E (x^2+z^2)\,\rho\, dV$$

$$I_z = \iiint_E (x^2+y^2)\,\rho\, dV$$

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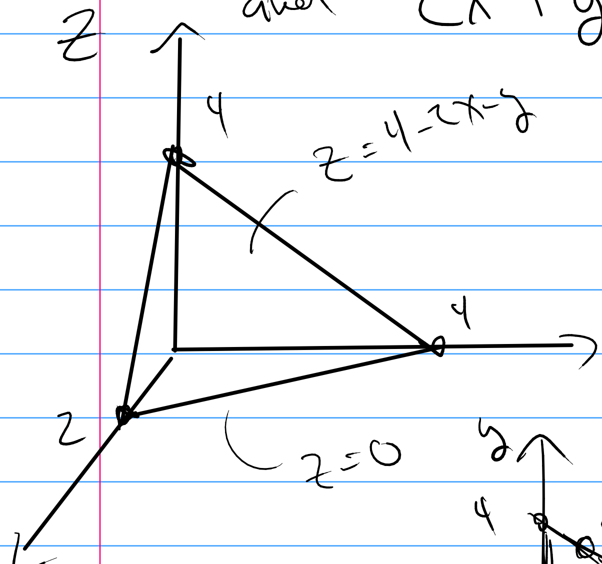
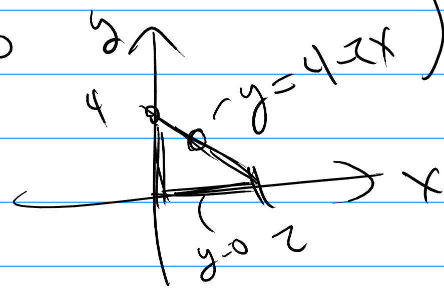
ex) Vol. of object enclosed by coord. planes and $2x+y+z=4$



$$V = \iiint_E (1)\,dV$$

$$\int_0^2 \left(\int_0^{4-2x} \left(\int_0^{4-2x-y} dz \right) dy \right) dx$$



$= ?$

$$= \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\, dy\, dx$$

$$= \text{''} \left(z \Big|_{z=0}^{z=4-2x-y} \right) \text{''}$$

$$= \int_0^2 \left(\int_0^{4-2x} (4-2x-y)\, dy \right) dx$$

$$= \int_0^2 \left(4y - 2xy - \tfrac{1}{2}y^2 \Big|_{y=0}^{y=4-2x} \right) dx$$

$$= \int_0^2 (4-2x)(4-2x) - \tfrac{1}{2}(4-2x)^2\, dx$$

$$= \frac{1}{2} \int_0^2 (4-2x)^2\, dx$$

$$u = 4-2x \qquad x=0 \to u=4$$

$$= \qquad du = -2\,dx \qquad x=2 \to u=0$$

$$= -\frac{1}{4} \int_4^0 u^2\, du = \frac{1}{4} \int_0^4 u^2\, du$$

$$= \frac{1}{4} \frac{1}{3} u^3 \Big|_0^4 = \boxed{\frac{16}{3}}$$

which is correct b/c Vol tetrahedron $= \frac{1}{3}$ (area base)(height) $= \frac{1}{3}(4)(4)$

$= 16/3$