Math 344

~~Q5~~

$$M = K \int_{\pi/6}^{5\pi/6} \left(\int_{9}^{18\sin\theta} dr \right) d\theta$$

$$M = K \int_{\pi/6}^{5\pi/6} (18\sin\theta - 9)\, d\theta$$

$$M = 9K \int_{\pi/6}^{5\pi/6} (2\sin\theta - 1)\, d\theta$$

$$= 9K \left(-2\cos\theta - \theta \right) \Big|_{\pi/6}^{5\pi/6}$$

$$= -9K \left[\left(\cancel{2} \overbrace{\cos 5\pi/6}^{-\sqrt{3}/\cancel{2}} + 5\pi/6 \right) - \left(2 \underbrace{\cos \pi/6}_{\sqrt{3}/2} + \pi/6 \right) \right]$$

$$= -9K \left[-\sqrt{3} + 2\pi/3 - \sqrt{3} \right]$$

$$M = \boxed{9K \left(2\sqrt{3} - 2\pi/3 \right)}$$

$$\bar{X} = \frac{K}{M} \int_{\pi/6}^{5\pi/6} \left(\int_{9}^{18\sin\theta} r\cos\theta \, dr \right) d\theta$$

$$= \frac{K}{M} \int_{\pi/6}^{5\pi/6} \cos\theta \left(\frac{1}{2} r^2 \Big|_{r=9}^{r=18\sin\theta} \right) d\theta$$

$$= \frac{K}{2M} \int_{\pi/6}^{5\pi/6} \cos\theta \left(18^2 \sin^2\theta - 9^2 \right) d\theta$$

$$= \frac{81K}{2M} \int_{\pi/6}^{5\pi/6} \cos\theta \left(4\sin^2\theta - 1 \right) d\theta$$

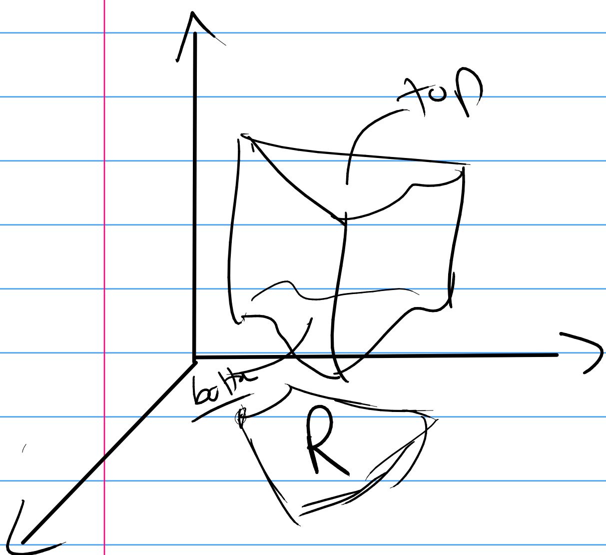
$$= \frac{81K}{2M} \int_{\pi/6}^{5\pi/6} \underbrace{4\cos\theta(\sin\theta)^2} - \underbrace{\cos\theta} \, d\theta$$

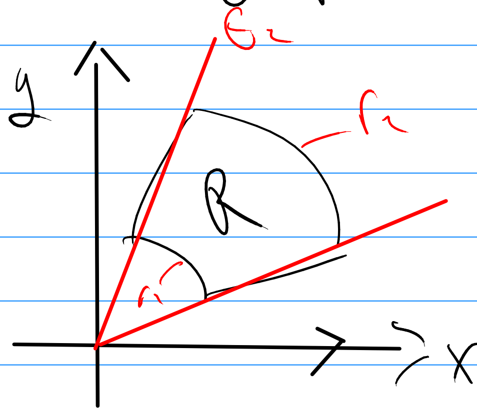
$$= \frac{81K}{2M} \left(\frac{4}{3} (\sin\theta)^3 - \sin\theta \right) \Big|_{\theta=\pi/6}^{\theta=5\pi/6}$$

$$= 0$$

$$\iiint_E f\,dV = \iint_R \left(\int_{\text{bottom } z=(\ )}^{\text{top } z=(\ )} f\,dz\right) dA$$

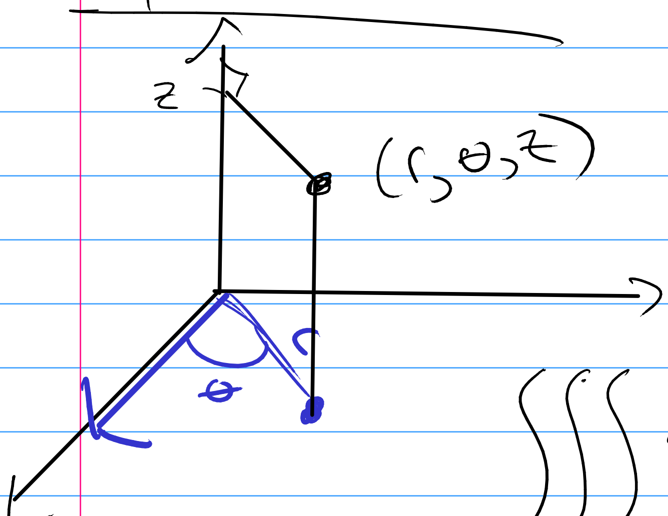
↑ region in xy plane





r, θ make a better idea on R

Cylindrical Coord. ← useful on objects E with a rotational axis.



$$\iiint_E f(x,y,z)\,dV$$

$$\iint_R \left(\int_{z=bot(r,\theta)}^{z=top(r,\theta)} f(r,\theta,z)\,dz\right) dA$$

$r\,dr\,d\theta$

Use

$x = r\cos\theta$ $\qquad r^2 = x^2+y^2$

$y = r\sin\theta$

$z = z$ $\qquad \tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$

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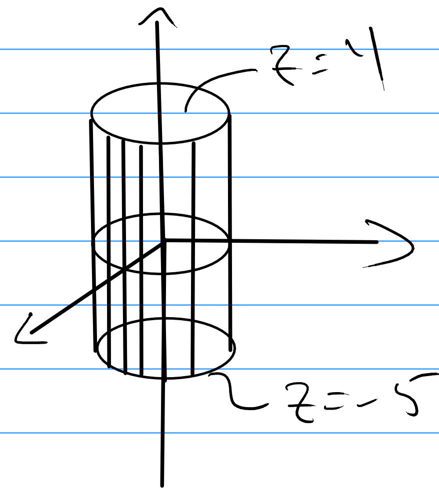
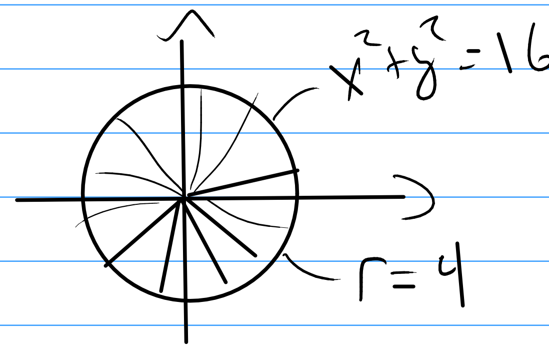
ex

$$\iiint_E \sqrt{x^2+y^2}\, dV$$



$$= \iint_R \left(\int_{z=-5}^{z=4} r\, dz \right) dA$$



$$= \int_0^{2\pi}\int_0^4\int_{-5}^4 r\cdot dz\cdot r\cdot dr\cdot d\theta$$

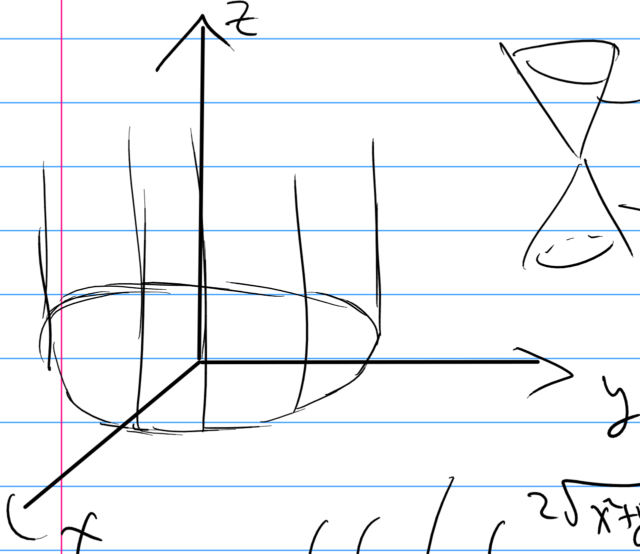
$$= \int_0^{2\pi} d\theta \int_0^4 r^2\, dr \int_{-5}^4 dz = (2\pi)\left(\frac{1}{\cancel{3}}4^3\right)(\cancel{9}^{\,3})$$

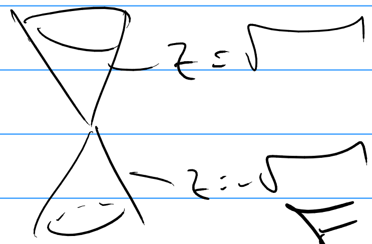
$$= 6\pi\cdot 64 = 6\cdot 64\pi$$

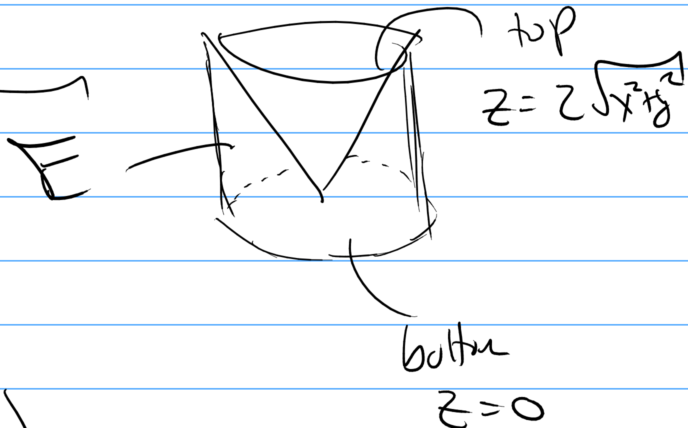
$$= \boxed{384\pi}$$

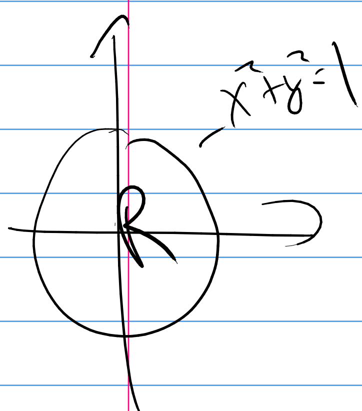
ex $\iiint_E x^2\,dV$ inside $x^2+y^2=1$ above $\boxed{z=0}$ xy plane

below $\boxed{z^2=4x^2+4y^2}$









$$\iint_R \left(\int_0^{2\sqrt{x^2+y^2}} x^2\,dz\right) dA$$

$$= \int_0^{2\pi}\int_0^1\int_0^{2r} \boxed{r^2\cos^2\theta}\,dz\;r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 r^3\cos^2\theta\left(\int_0^{2r} dz\right)dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 2r^4\cos^2\theta\,dr\,d\theta$$

$$= \int_0^{2\pi}\cos^2\theta\,d\theta\int_0^1 2r^4\,dr = \text{finish}$$