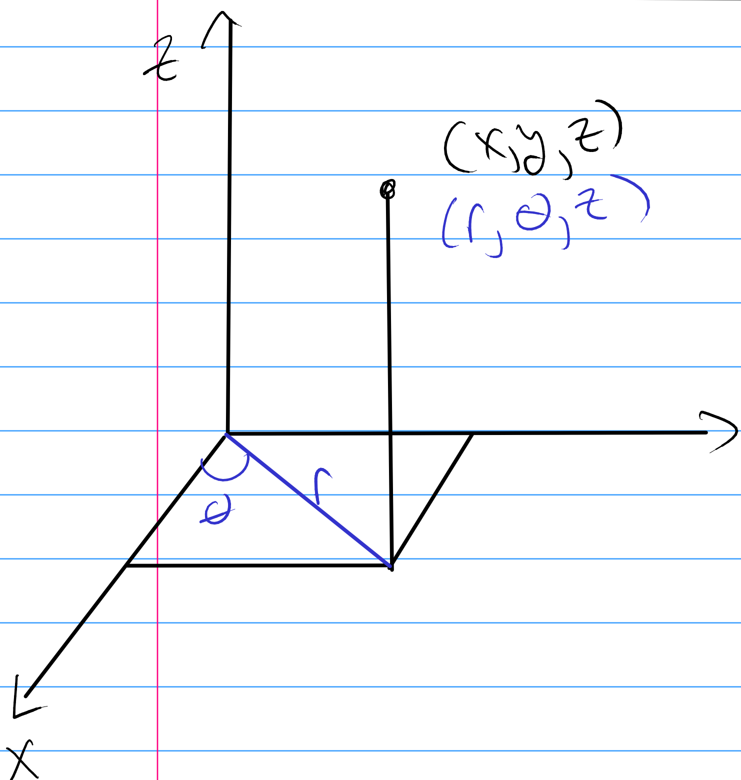
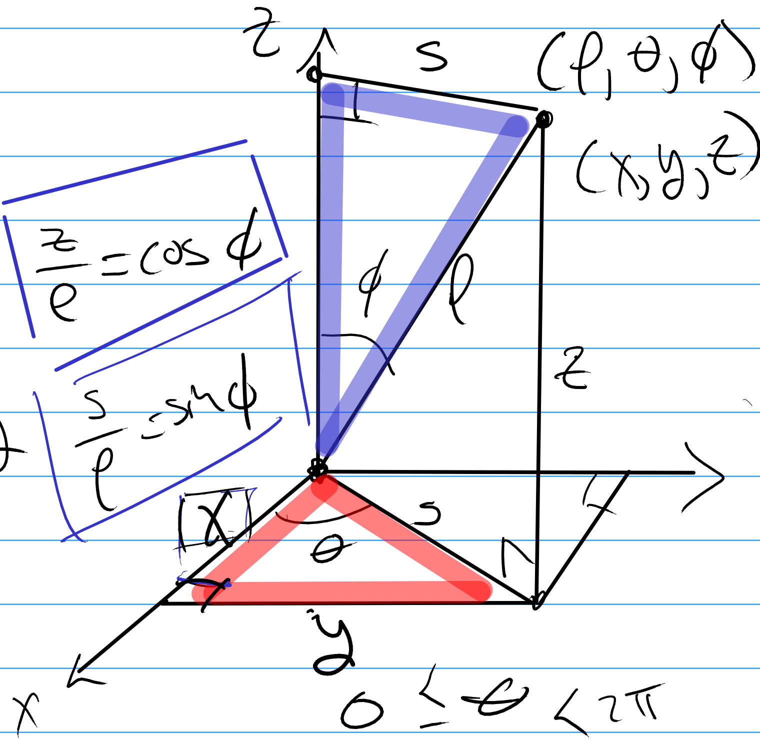
Math 344





$$\boxed{\frac{z}{\rho} = \cos\phi}$$

$$\boxed{\frac{s}{\rho} = \sin\phi}$$

$0 \le \theta < 2\pi$

$0 \le \rho$

$0 \le \phi \le \pi$

$$\boxed{\begin{aligned} z &= \rho\cos\phi \\ x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ x^2+y^2+z^2 &= \rho^2 \end{aligned}}$$

$$s = \rho\sin\phi$$

$$\boxed{\frac{x}{s} = \cos\theta}$$

$$\boxed{\frac{y}{s} = \sin\theta}$$

$$\iiint_E f\, dV$$

$$dV = (\Delta\rho)(\rho\Delta\phi)(\rho\sin\phi\,\Delta\theta)$$

$$dV = \rho^2\sin\theta\;\boxed{d\rho\, d\theta\, d\phi}$$

$$\iiint_E f(x,y,z)\,dV$$

$$= \iiint_E \left[f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)\,\rho^2\sin\phi\right] d\rho\, d\theta\, d\phi$$

in ρ, θ, ϕ

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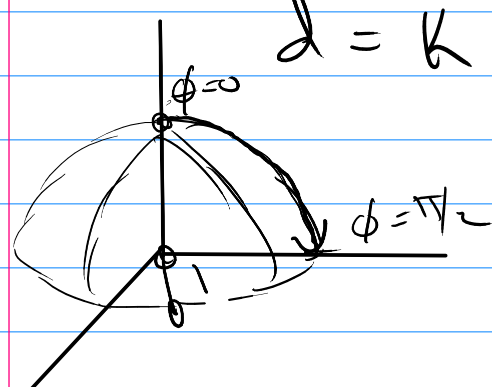
ex) find mass of upper hemisphere of radius 1

density $\propto$ distance from origin

$d = k\rho$ ← let $k=1$



$0 \le \rho \le 1$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi/2$

$$\iiint_E \rho \, (\underbrace{\rho^2 \sin\phi \, d\rho \, d\theta \, d\theta}_{dV})$$

$$= \int_{\phi=0}^{\phi=\pi/2} \int_{\theta=0}^{\theta=2\pi} \int_{\rho=0}^{\rho=1} \rho^3 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} \sin\phi \, d\phi \cdot \int_0^{2\pi} d\theta \cdot \int_0^1 \rho^3 \, d\rho$$

$$= \left(-\cos\phi \Big|_0^{\pi/2}\right)(2\pi)\left(\frac{1}{4}\right)$$

$$= (1)(2\pi)\left(\frac{1}{4}\right) = \boxed{\pi/2}$$

$\bar{x} = 0 \quad \bar{y} = 0$ (by uniformity)

$$\bar{z} = \frac{1}{m} \iiint \underbrace{\boxed{z}}_{\rho\cos\phi} \cdot \underbrace{\boxed{\text{density}}}_{\rho} \, \underbrace{\boxed{dV}}_{\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi}$$

$$= \frac{1}{m} \iiint_E \rho^4 \boxed{\cos\phi \sin\phi} \, d\rho \, d\theta \, d\phi$$

$$\bar{z} = \frac{1}{M} \underbrace{\int_0^{\pi/2} \cos\phi \sin\phi \, d\phi}_{\text{use sub.}} \int_0^{2\pi} d\theta \int_0^1 \rho^4 \, d\rho$$

$$= \frac{1}{\pi/2} \left(\frac{1}{2} \right) \left(2\cancel{\pi} \right) \left(\frac{1}{5} \right)$$

$$= 2/5$$

center of mass $= \boxed{\left(0, 0, 2/5\right)}$