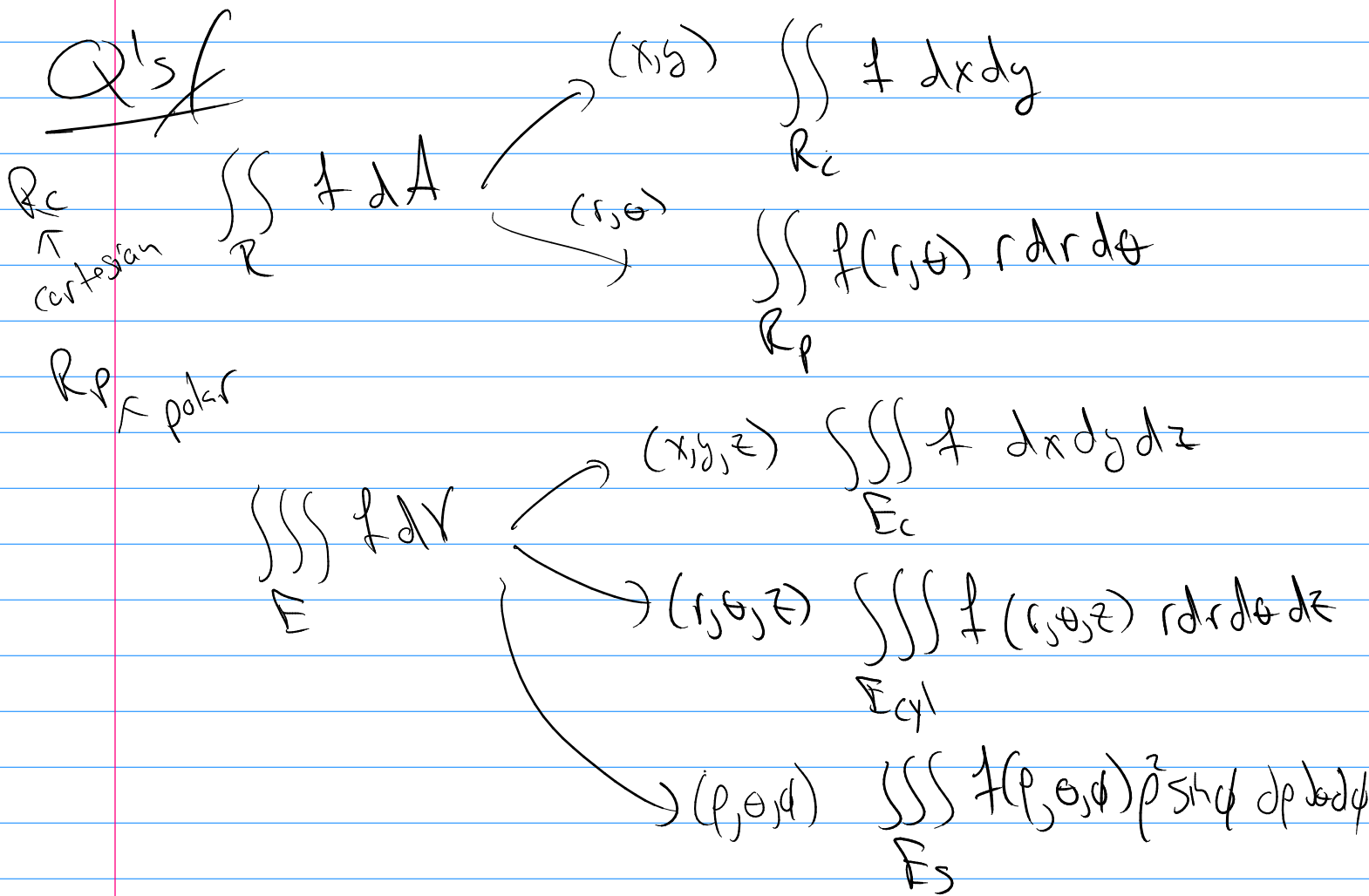


Math 349



IB

$$\int_c^b f(x) \, dx = \int_c^d f(x(u)) \frac{dx}{du} \, du$$

$x = x(u)$

Qx

$$\int_0^1 x \sqrt{x^2+1} dx = \int_1^2 \frac{1}{2} u^{1/2} du$$

$$u = x^2 + 1 \rightarrow \sqrt{x^2+1} = \sqrt{u-1}$$

$$du = 2x dx$$

1D

$$\int_a^b f(x) dx \rightarrow \int_c^d f(x(u)) \frac{dx}{du} du$$

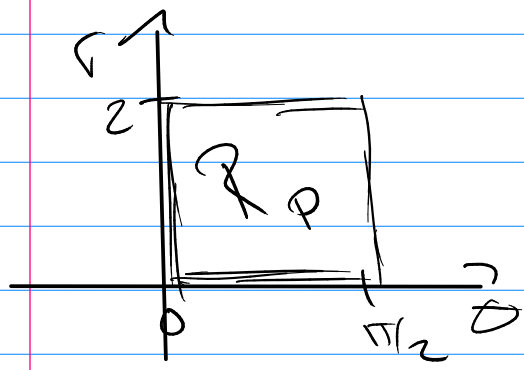
$x = x(u) \leftarrow$ Transformation

2D

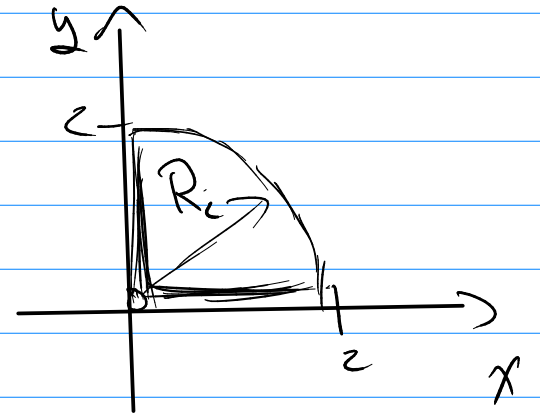
$$\iint_{R_c} f(x,y) dA = \iint_{R_p} f(x(r,\theta), y(r,\theta)) r dr d\theta$$

$$x = x(r,\theta) = r \cos \theta$$

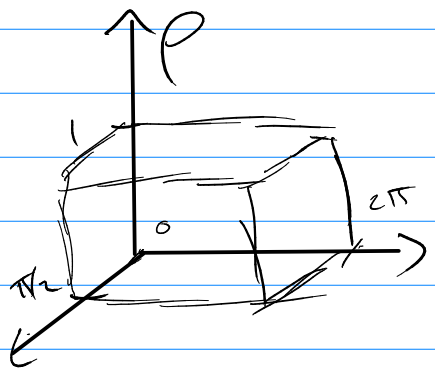
$$y = y(r,\theta) = r \sin \theta$$



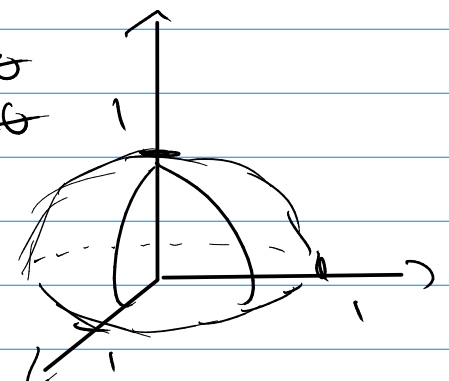
$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$



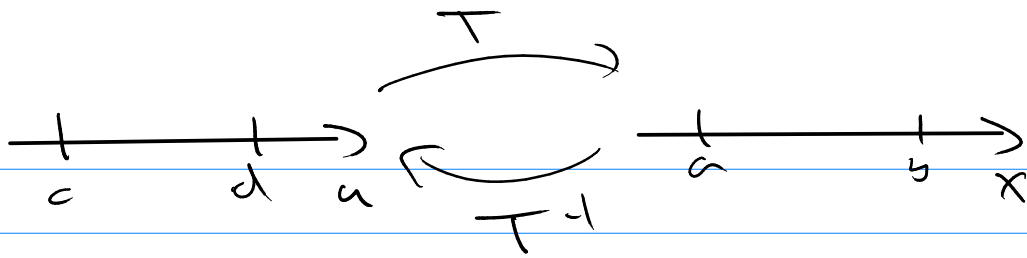
3D



$$\begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases}$$

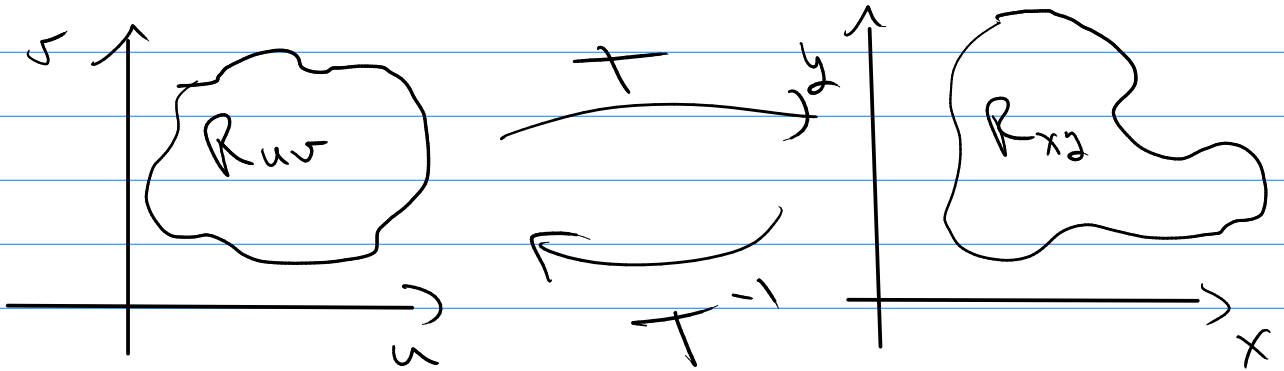


1D



$$T: x = x(u)$$

2D

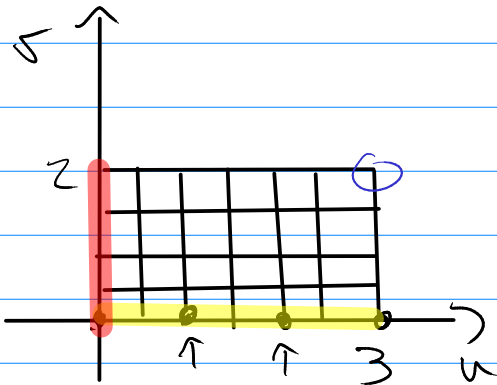


$$T: \begin{aligned} x &= x(u, v) \\ y &= y(u, v) \end{aligned}$$

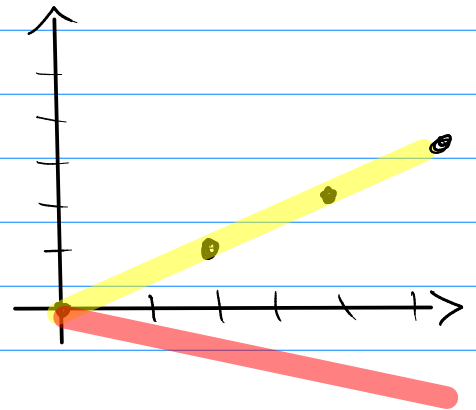
ex

R_{uv}

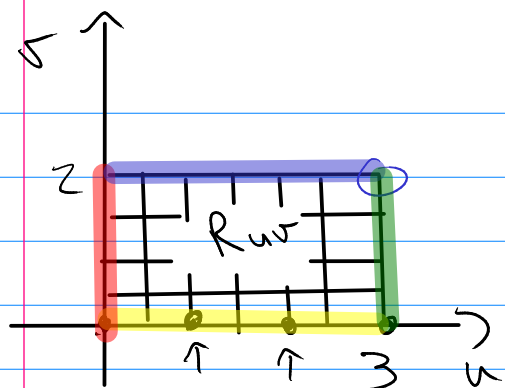
$$S = \{ (u, v) \mid 0 \leq u \leq 3, 0 \leq v \leq 2 \}$$



$$T \begin{aligned} x &= 2u + 3v \\ y &= u - v \end{aligned}$$



$$u=0 \quad \left. \begin{aligned} x &= 3v \\ y &= -v \end{aligned} \right\} y = -x/3$$

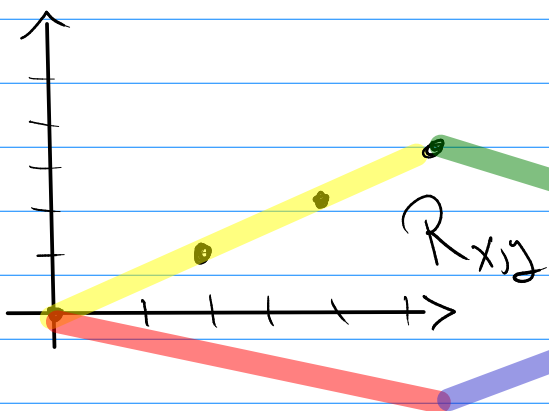


$$\begin{matrix} \xrightarrow{T} \\ \boxed{\begin{matrix} x = 2u + 3v \\ y = u - v \end{matrix}} \end{matrix}$$

$$\textcircled{dA} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$(u, v) = (3, 2)$$

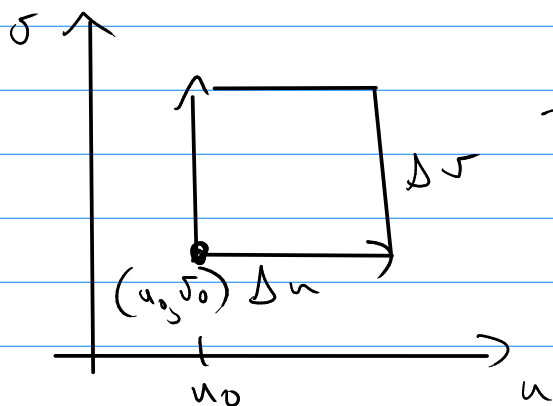
$$\xrightarrow{T} (x, y) = (12, 1)$$



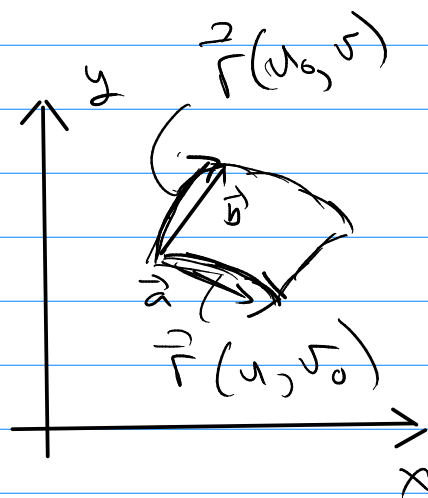
$$\textcircled{dA} = dx dy$$

dA in x, y is $dx dy$

Problem: dA in u, v ?



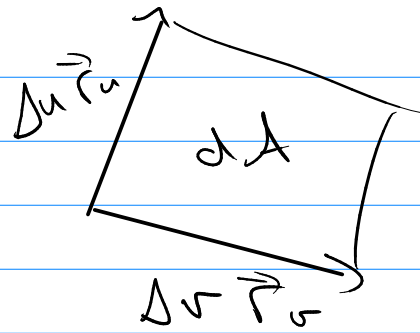
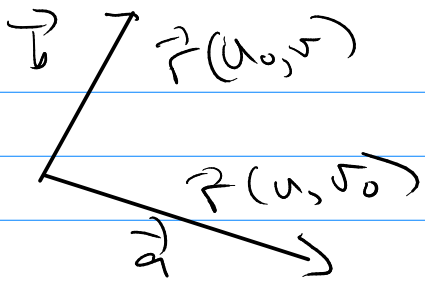
$$\xrightarrow{T} \begin{pmatrix} x(u, v) \\ y(u, v) \end{pmatrix}$$



$$\vec{r} = \langle x(u, v), y(u, v) \rangle$$

$$\vec{r}_u = \langle x_u, y_u \rangle$$

$$\vec{r}_v = \langle x_v, y_v \rangle$$



$$\vec{a} \approx \Delta u \vec{r}_u$$

$$\vec{b} \approx \Delta v \vec{r}_v$$

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$dA = \left| \vec{r}_u \times \vec{r}_v \right| du dv$$

$$\vec{r} = \langle x(u, v), y(u, v) \rangle$$

Corresponds

factor to

add to $du dv$