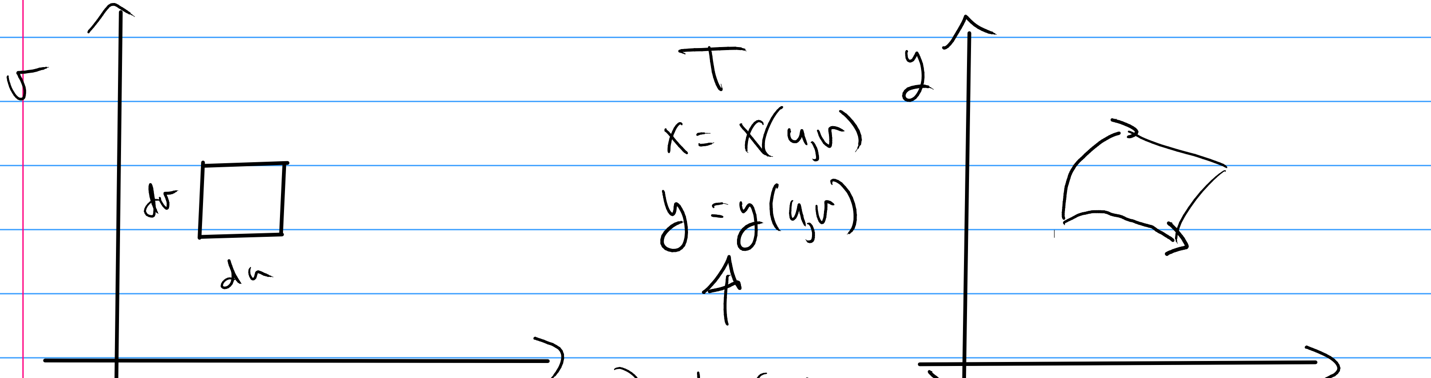
# Math 344




$$\vec{r} = \langle x(u,v), y(u,v) \rangle$$

$$dA = |\vec{r}_u \times \vec{r}_v| \, du\, dv$$

$$\vec{r}_u = \langle x_u, y_u \rangle$$

$$\vec{r}_v = \langle x_v, y_v \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle x_u, y_u, 0 \rangle \times \langle x_v, y_v, 0 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_u & y_u & 0 \\ x_v & y_v & 0 \end{vmatrix} = \left\langle 0, 0, \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} \right\rangle$$

$$= (x_u y_v - x_v y_u) \cdot \vec{k}$$

So $$dA = |\boxed{\vec{r}_u \times \vec{r}_v}| \, du\, dv$$

$$= \left| \left\langle 0, 0, \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} \right\rangle \right| du\, dv$$

$$dA = \left| \det\begin{pmatrix} x_u & y_u \\ x_v & y_v \end{pmatrix} \right| du\,dv$$

abs value det

$$dA = \text{abs}\left( \det\begin{pmatrix} x_u & y_u \\ x_v & y_v \end{pmatrix} \right) du\,dv$$

**Fund:** $\displaystyle\iint_{R_{xy \text{ plane}}} f(x,y)\, dA$

$$= \iint_{R_{uv \text{ plane}}} f(x(u,v), y(u,v)) \overbrace{\left| \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} \right| du\,dv}^{dA}$$

$$T: \langle x(u,v), y(u,v) \rangle$$

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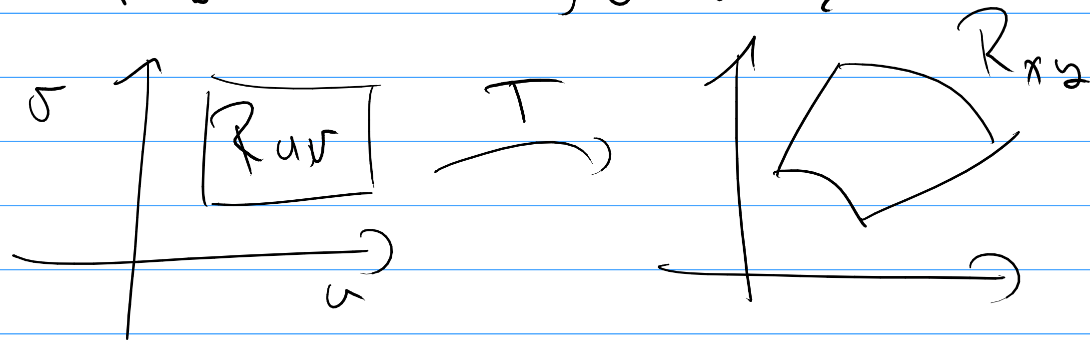
**Def:** $$\text{Jacobian} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

**Notation:** $$\text{Jacobian} = \frac{\partial(x,y)}{\partial(u,v)}$$

$$\rightarrow \iint_{R_{xy}} f\,dA = \iint_{R_{uv}} f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$

$$T : \langle x(u,v), y(u,v) \rangle$$

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ex) $T : \quad x = r\cos\theta, \quad y = r\sin\theta$

$$\iint_{R_{xy}} f\,dA = \iint_{R_{r\theta}} f(r\cos\theta, r\sin\theta)\, dA$$

$$\left| \frac{\partial(x,y)}{\partial(r,\theta)} \right| dr\,d\theta$$

$T$: $x = r\cos\theta$
$y = r\sin\theta$

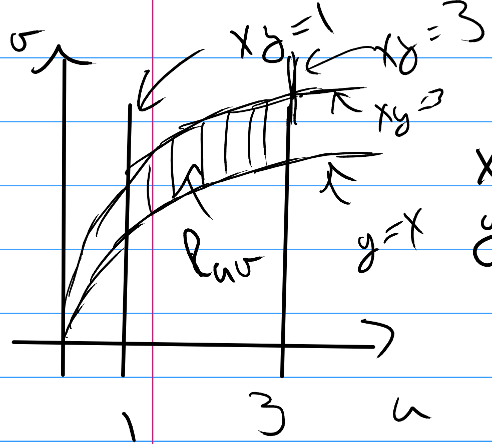
$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & y_r \\ x_\theta & y_\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix}$$

$$= (r\cos^2\theta) - (-r\sin^2\theta)$$

$$= r(\cos^2\theta + \sin^2\theta) = r$$

$$dA = |r|\,dr\,d\theta = r\,dr\,d\theta$$

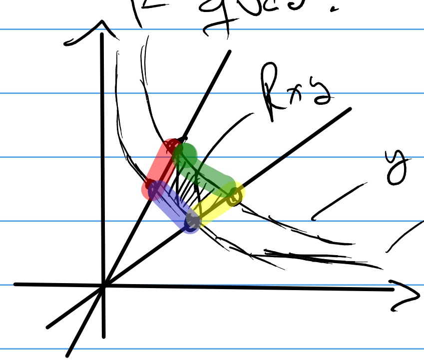
(ex) $\iint_{R_{xy}} y^2 \, dA$ $R_{xy}$ is bounded by $y=x$, $y=3x$, $xy=1$, $xy=3$, and in 1st quad.




T

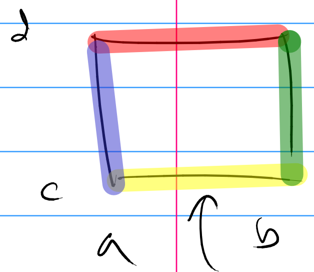
$x = u/v$

$y = v$




$y = \frac{1}{x}$

$y = \frac{3}{x}$




$xy = 1$ $\quad$ $xy = 3$

$\left(\frac{u}{v}\right)(v) = 1$ $\quad$ $u = 3$

$u = 1$

$y = x$ $\quad$ $y = 3x$

$v = \frac{u}{v}$ $\quad$ $v = 3\frac{u}{v}$

$v^2 = u$ $\quad$ $v^2 = 3u$

$v = \sqrt{u}$ $\quad$ $v = \sqrt{3u}$

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$$\int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} f(u,v) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dv\, du$$

$$= \int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} v^2 \left| \frac{\partial(x,y)}{\partial(u,v)} \right| dv\, du$$

Jacobian?

$x = u/v$

$y = v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & 0 \\ -\frac{u}{v^2} & 1 \end{vmatrix} = \frac{1}{v}$$

$$\int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} v^{\cancel{2}} \frac{1}{\cancel{v}} \, dv \, du = \underline{\underline{\text{Finish}\ldots}}$$

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