Math 3414

~~Q's~~ 12.8

$y = 2x - 3$ $\quad$ $y = 2x + 3$

$-2x + y = -3$ $\quad$ $-2x + y = 3$ $\quad$ let $u = -2x + y$

$u = -3$ $\quad$ $u = 3$

$y = 3 - x$ $\quad$ $y = 5 - x$

$x + y = 3$ $\quad$ $x + y = 5$ $\quad$ let $v = x + y$

$v = 3$ $\quad$ $v = 5$

$$T^{-1}\begin{cases} u = -2x + y \\ v = x + y \end{cases}$$

T? : $x = x(u, v)$

$y = y(u, v)$

$$u = -2x + y$$
$$-(v = x + y)$$
$$u - v = -3x$$

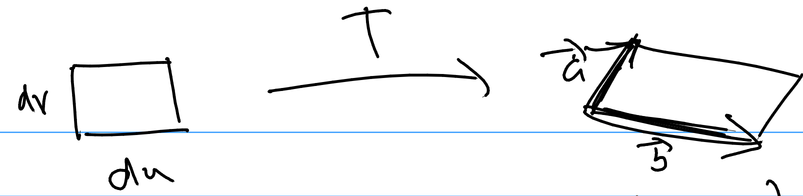
$$\boxed{x = \frac{v - u}{3}}$$

let $y = v - x$

$$y = v - \frac{1}{3}v + \frac{1}{3}u$$

$$\boxed{y = \frac{2v + u}{3}}$$

$\boxed{2D}$



$T: \vec{r} = \langle x(u,v), y(u,v)\rangle \quad dA = |\vec{a} \times \vec{b}|$

$$dA = |\vec{r}_u \times \vec{r}_v| \, du\,dv$$

$$dA = \left| \left\langle 0, 0, \boxed{\begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix}} \right\rangle \right| du\,dv$$

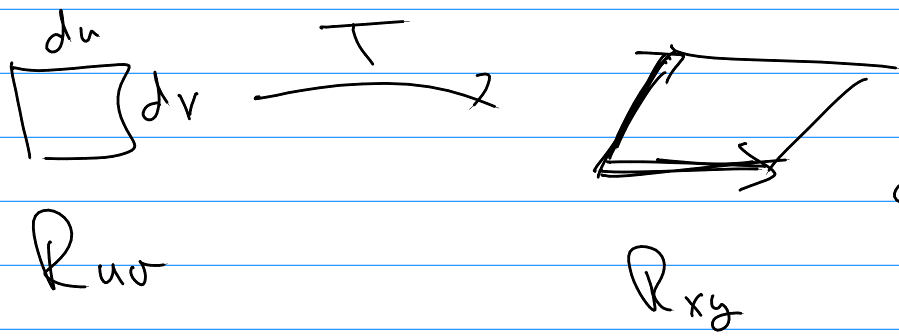
Lin. Alg:
$$\begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

$$\det(A) = \det(A^T)$$

Why? $x(u,v) \rightarrow \nabla x = \langle x_u, x_v \rangle$

Jacobian:
$$\underbrace{\begin{array}{cc} \begin{vmatrix} \nabla x, & \nabla y \\ \uparrow & \uparrow \\ \text{col} & \text{col} \end{vmatrix} \end{array}}_{\text{me}} = \underbrace{\begin{vmatrix} \nabla x \\ \nabla y \end{vmatrix}}_{\text{book}}$$

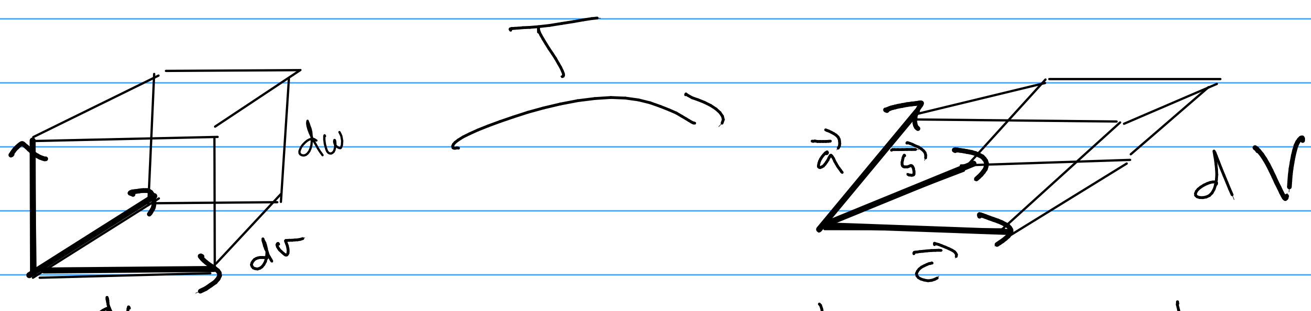
$\boxed{2D}$ $$\iint\limits_{R_{xy\text{-plane}}} f\,dA = \iint\limits_{R_{uv\text{-plane}}} f(\underbrace{x(u,v),y(u,v)}_{\uparrow\, T})\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$



$$dA = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$\boxed{3D}$ $$\iiint\limits_{R_{xyz\text{-space}}} f\,dV = \iiint\limits_{R_{uvw\text{-space}}} f(?)\,(?)$$

$$T : \langle x(u,v,w),\ y(u,v,w),\ z(u,v,w) \rangle$$



$$|\vec{a}\cdot(\vec{b}\times\vec{c})|$$

$$\to \vec{a}\cdot(\vec{b}\times\vec{c}) = |\vec{a},\vec{b},\vec{c}| = \begin{vmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{vmatrix}$$

$$\rightarrow dV = \left| \begin{vmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{vmatrix} \right| du\,dv\,dw$$

$$dV = \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du\,dv\,dw$$

<u>book:</u>

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \nabla x \\ \nabla y \\ \nabla z \end{vmatrix}$$

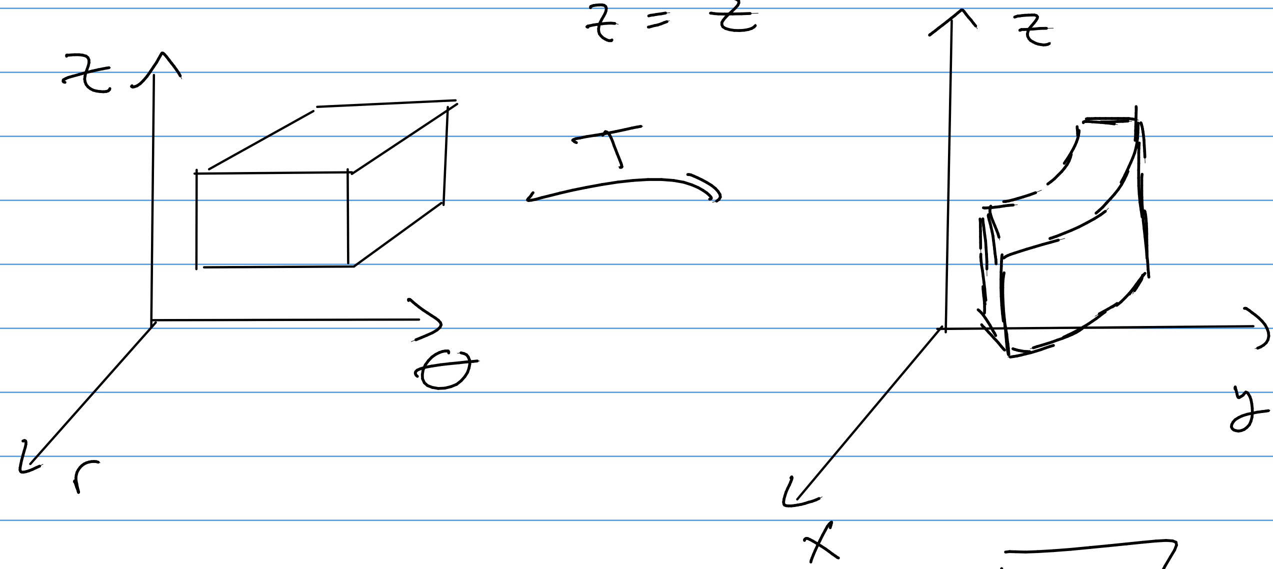
$$\text{So} \quad \iiint_{E_{xyz}} f\,dV = \iiint_{E_{uvw}} f(x(u,v,w), y(u,v,w), z(u,v,w)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du\,dv\,dw$$

$$T : \vec{r} = \langle x(u,v,w), y(u,v,w), z(u,v,w) \rangle$$

Different symbols.

$$\iiint_{E_{uvw}} f(\vec{r}(u,u,v)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du\,dv\,dw$$

$$\boxed{\text{Cylindrical}} \qquad T: \begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned}$$



$$\iiint_{E_{xyz}} f \, dV = \iiint_{E_{r\theta z}} f(r\cos\theta, r\sin\theta, z) \boxed{dV}$$

$$dV = \left| \frac{\partial(x,y,z)}{\partial(r,\theta,z)} \right| dr \, d\theta \, dz$$

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$$\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{vmatrix} \nabla x \\ \nabla y \\ \nabla z \end{vmatrix} = \begin{vmatrix} x_r & x_\theta & x_z \\ y_r & y_\theta & y_z \\ z_r & z_\theta & z_z \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 0 \cdot \left| \cdots \right| + (-1) \cdot 0 \cdot \left| \cdots \right| + 1 \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r$$

$$\to \iiint_{E_{r\theta z}} f(r,\theta,z)\, r\,dr\,d\theta\,dz$$

try: Spherical.

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)}$$