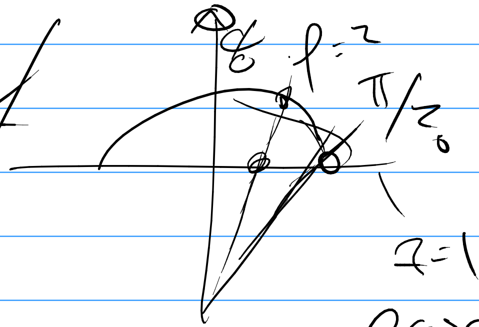
Math 344

Q's



$\rho\cos\phi = 1 \quad \rho = \frac{1}{\cos\phi}$

Line Integrals

$$\int_C f\,ds = \int_a^b f(\vec{r})\,|\vec{r}\,'|\,dt$$

$$\int_C f\,dx = \int_a^b f(\vec{r})\,x'\,dt$$

etc.

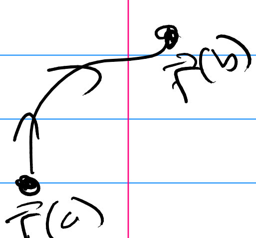
$$\int_C \vec{F}\cdot d\vec{r} = \int_C \vec{F}\cdot\vec{T}\,ds = \int_a^b \vec{F}(\vec{r})\cdot\vec{r}\,'\,dt$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C P\,dx + Q\,dy + R\,dz$$

$$\vec{F} = \langle P, Q, R\rangle$$

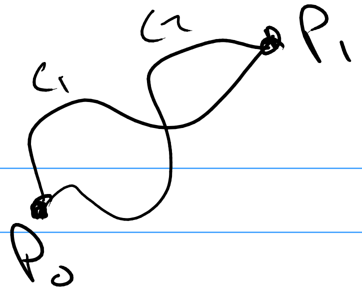
Fund. Thm

if $\vec{F} = \nabla f$



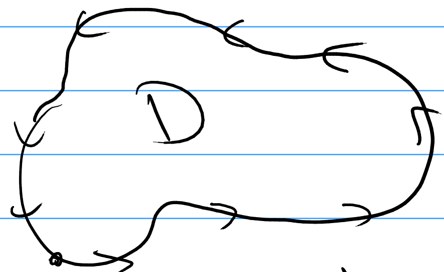
$$\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

13.3



$$\int_{C_i} \nabla f \cdot d\vec{r} = f(P_1) - f(P_0)$$

13.4



C ← positive orientation

<u>Q:</u> $\iint_D (\ )\,dA \overset{?}{=} \int_C (\ )\,ds$

<u>Note:</u> $\oint_C$ line integral in pos. orientation

∂D ← boundary of D in pos. orientation

Green's Thm

C is pos-orient, piece-wise smooth, simple closed curve and D is the region bounded by C $(C = \partial D)$

Then if P, Q have cont. partials in an open region of D

$$\iint_D (Q_x - P_y)\,dA = \oint_{\substack{\partial D \\ \shortparallel \\ C}} P\,dx + Q\,dy$$

$$\vec{F} = \langle P, Q \rangle$$

Green's Thⁿ

$$\iint_D (Q_x - P_y)\, dA = \int_{\partial D} P\,dx + Q\,dy$$

$$= \int_{\partial D} \vec{F} \cdot d\vec{r}$$

Two Uses:

① $$\int_{\partial D} \underbrace{\vec{F} \cdot d\vec{r}}_{\text{hard!}} = \int_{\partial D} P\,dx + Q\,dy$$

$$= \iint_D \underbrace{(Q_x - P_y)\, dA}_{\text{hopefully easier!}}$$

② $$\underbrace{\iint_D (Q_x - P_y)\, dA}_{\text{Hard?}} = \int_{\partial D} P\,dx + Q\,dy$$

Special Case: Find the area of D

$$\iint_D \underbrace{(1)}_{Q_x - P_y} dA = \int_{\partial D} P\,dx + Q\,dy$$

a) $Q = x \quad P = 0 \rightarrow Q_x - P_y = 1$

$$\boxed{\text{Area}(D) = \int_{\partial D} x\,dy}$$

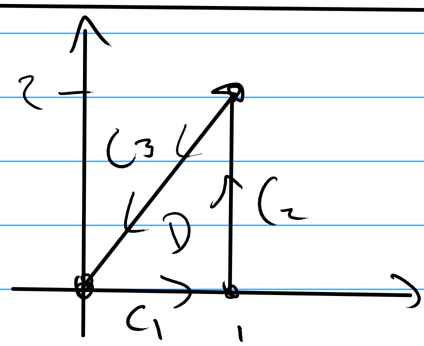
b) $Q = 0 \quad P = -y \rightarrow Q_x - P_y = 1$

$$\text{Area}(D) = -\int_{\partial D} y\,dx$$

c) $Q = \frac{1}{2}x \quad P = -\frac{1}{2}y \rightarrow Q_x - P_y = 1$

$$\text{Area}(D) = \frac{1}{2}\int_{\partial D} x\,dy - y\,dx$$

Ex

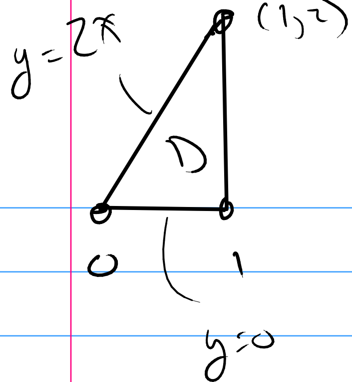


$$\int_{\partial D} xy\,dx + x^2y^3\,dy$$

Green's $\displaystyle\int_{\partial D} P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$

w/o Green's $\quad \partial D \rightarrow \displaystyle\int_{\partial D} = \int_{C_1} + \int_{C_2} + \int_{C_3}$

w/ Green's $\displaystyle\int_{\partial D} xy\,dx + x^2y^3\,dy = \iint_D (2xy^3 - x)\,dA$



$$\iint_D (2xy^3 - x)\, dA$$

$$= \int_0^1 \int_0^{2x} (2xy^3 - x)\, dy\, dx$$

$$= \int_0^1 \left(\frac{1}{2}xy^4 - xy \Big|_{y=0}^{y=2x} \right) dx$$

$$= \int_0^1 (8x^5 - 2x^2)\, dx$$

$$= \frac{4}{3}x^6 - \frac{2}{3}x^3 \Big|_0^1 = \frac{4}{3} - \frac{2}{3} = \boxed{\frac{2}{3}}$$