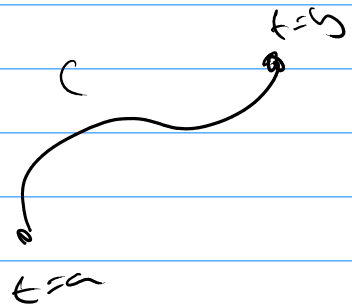
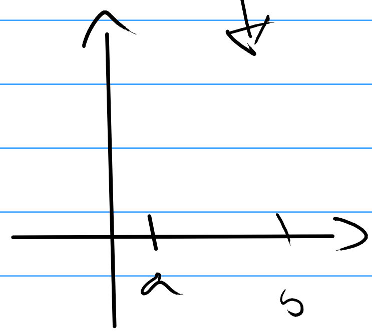
Math 344

$\boxed{2D}$ $\vec{F} = \langle P, Q \rangle$ $\boxed{3D}$ $\vec{F} = \langle P, Q, R \rangle$

Line Integral

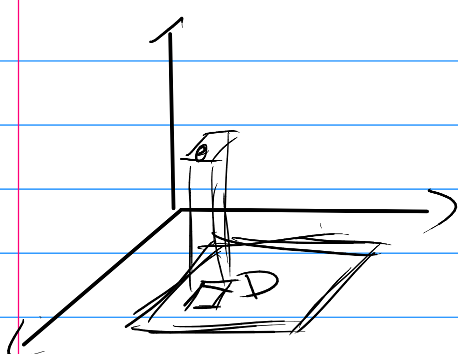


$$\int_C f\,ds$$

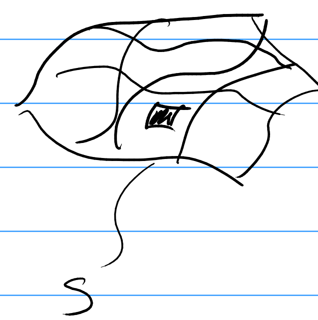


$$\int_{x=a}^{x=b} f\,dx$$

$\boxed{\text{Calc 1}}$



$\boxed{3D}$



① Surface in 3D

$\boxed{\text{Cartesian}}$ → Explicit surface: $z = f(x,y)$

↳ Implicit: $F(x,y,z) = 0$

$\boxed{\text{plots}}$ $\leftrightarrow$ set of all points that make an eqn <u>true</u>.

$\boxed{\text{ex}}$ $F(x,y,z)=0$

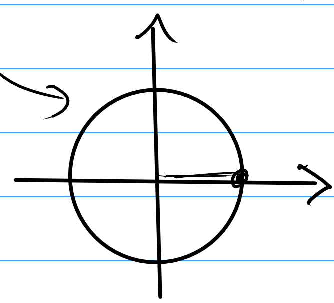
$$x^2+y^2+z^2-a^2=0$$

$\boxed{\text{Q}}$ Can you find something that generates the same pts?

$\boxed{\text{ex}}$ $x^2+y^2=1 \leftarrow S$

Implicit

$\boxed{\text{2D}}$

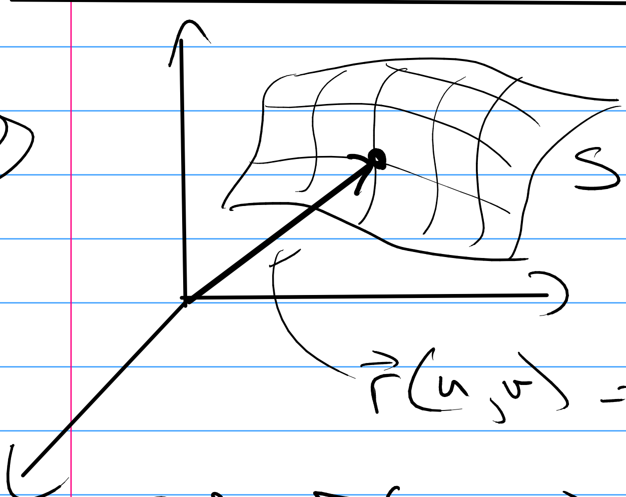


$\boxed{\text{parametric eqns}}$

$$\vec{r}=\langle x(t),\, y(t)\rangle$$

$$\begin{cases} x(t)=\cos(t) \\ y(t)=\sin(t) \end{cases} \qquad \underline{\underline{0\le t<2\pi}}_{\;D}$$

$\boxed{\text{3D}}$



$(u,v)\in D$

$$\left.\begin{matrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{matrix}\right\}$$ parametric eqns that make $\boxed{F(x,y,z)=0}$

$$\vec{r}(u,v)=\langle x(u,v),\, y(u,v),\, z(u,v)\rangle$$

S: $F(x,y,z)=0$ (Implicit form)

$\vec{r}(u,v)$ is a parametric surface.

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plots? CAS

hand? $\vec{r}(u,v) = \langle x, y, z \rangle$

let $\boxed{u = \text{const}}$

$\hookrightarrow \vec{r}_{u_{con}}(v) = \boxed{\text{parametric curve}}$

ex $x = (2 + \sin v)\boxed{\cos u}$

$y = (2 + \sin v)\boxed{\sin u}$

$z = u + \cos v$

$\left.\begin{array}{l} \\ \\ \end{array}\right\} \vec{r}$

let $\underline{\underline{v = \text{const.}}}$ ex $v = 0$

$x = 2\cos u$

$y = 2\sin u$

$z = u + 1$

Finding $\vec{r}(u,v)$

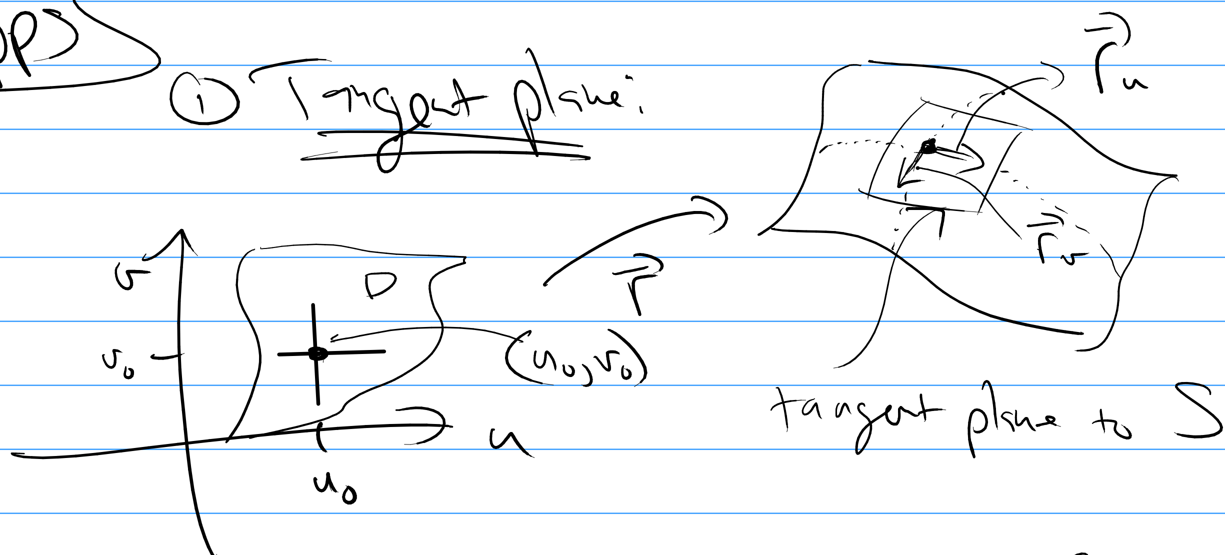
① <u>explicit</u> $z = f(x,y)$

$$x = u$$
$$y = v$$
$$z = f(u,v)$$

② <u>Implicit?</u> → spherical identities
→ cylindrical identities

Apps

① <u>Tangent plane:</u>



tangent plane to S

use $\vec{r}_u \times \vec{r}_v$ for normal (@ u_0, v_0)

use $\vec{r}(u_0, v_0) = $ pt. $= (x_0, y_0, z_0)$

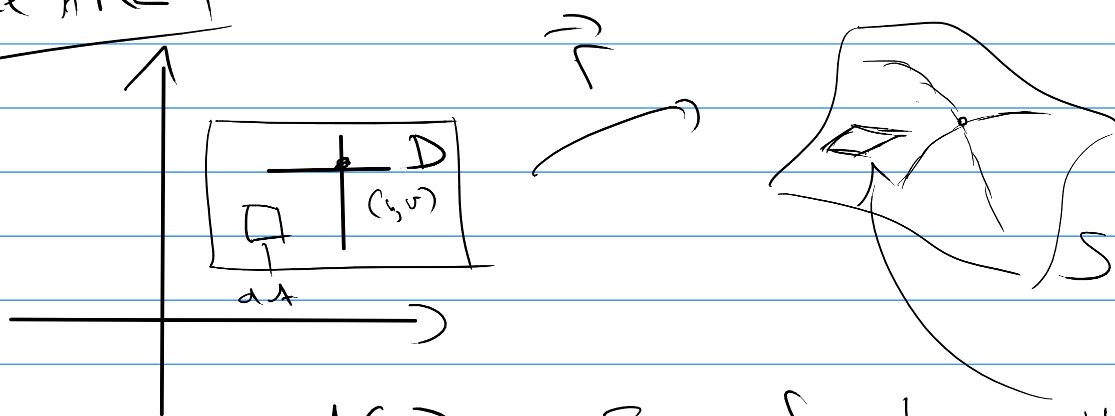
ex @ (u_0, v_0) pt $x(u_0,v_0) = x_0$
$y(u_0,v_0) = y_0$
$z(u_0,v_0) = z_0$

$$\vec{r}_u \times \vec{r}_v \Big|_{(u_0,v_0)} = \vec{n} = \langle a,b,c \rangle$$

Cont

$$\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$$

Surface Area



$A(S)$ = Sum of the parallelograms

area parallelograms $= |r_u \times r_v| \underbrace{du\,dv}_{dA}$

in u,v

$$\boxed{A(S)=\iint_{D_{u,v}} |\vec{r}_u\times\vec{r}_v|\, dA}$$

Note: $z=f(x,y)$ ← explicit function

Let $u=x$, $v=y$ (x-y space)

$$\rightarrow A(S)=\iint_D \boxed{\sqrt{1+f_x^2+f_y^2}}\, dA$$