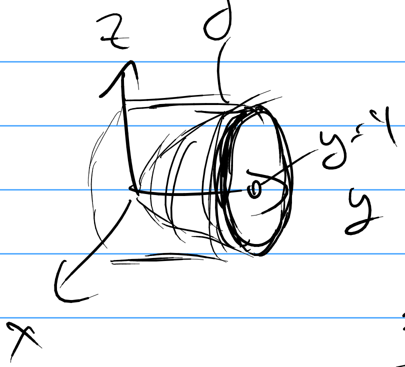
Math 344

Q's $\iint_S \boxed{y}\, dS$ S: paraboloid $y = x^2+z^2$ inside $x^2+z^2=4$

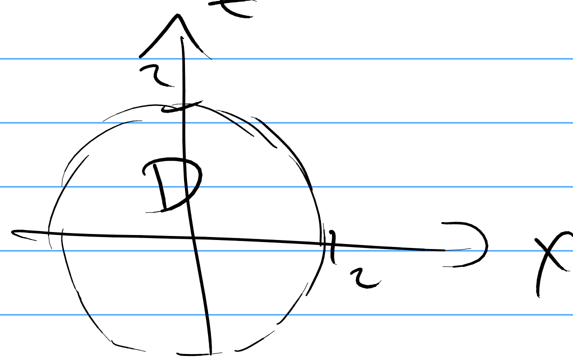
$$\iint_S f\, dS = \iint_D \left(f(\vec r)\,|\vec r_u \times \vec r_v|\right) dA$$

parameterize

S : $\vec r(u,v)$ $(u,v) \in D$



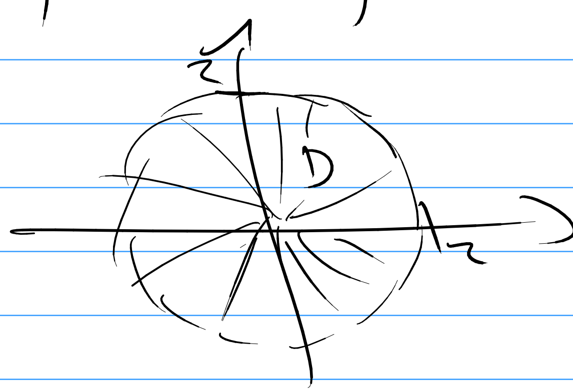
S: $y = \boxed{x^2+z^2}$ from $y=0$ to $y=4$



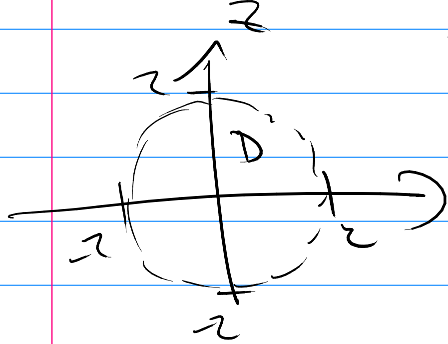
param's (x, z)

$$\vec r = \langle x,\ x^2+z^2,\ z \rangle$$

over D



$$\iint_S (y)\, dS = \iint_D (x^2+z^2)\,|\vec r_x \times \vec r_z|\, dA$$



$$\vec r = \langle x, x^2+z^2, z\rangle$$

$$|\vec r_x \times \vec r_z| = \left|\begin{vmatrix} i & j & k \\ 1 & 2x & 0 \\ 0 & 2z & 1 \end{vmatrix}\right|$$

$$= |\langle 2x, -1, 2z\rangle|$$

$$= \left((2x)^2 + 1 + (2z)^2\right)^{1/2}$$

$$\iint_S y\, dS = \iint_D \left[(x^2+z^2)\sqrt{4x^2+4z^2+1}\right] dA$$

Cartesian $= \displaystyle\int_{-2}^{2}\left(\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2+z^2)\sqrt{4x^2+4z^2+1}\, dz\right) dx$

Polar $= \displaystyle\int_0^{2\pi}\int_0^{2} r^2\sqrt{4r^2+1}\; r\,dr\, d\theta$

$$= \underbrace{\int_0^{2\pi} d\theta}_{2\pi} \int_0^2 r^3 \underbrace{\sqrt{4r^2+1}}_{\text{sub}}\, dr = \text{Finish}$$

13.7 $\iint_S 1 \, dS$

scalar

App $\iint_S \boxed{\vec{F} \cdot \vec{n}} \, dS = \iint_S \vec{F} \cdot d\vec{S}$

normal of S

$$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$$

$$\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_D \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) \, dA$$

App ① Electric flux

$$\iint_S \vec{E} \cdot d\vec{S}$$

② Rate of Heat flow

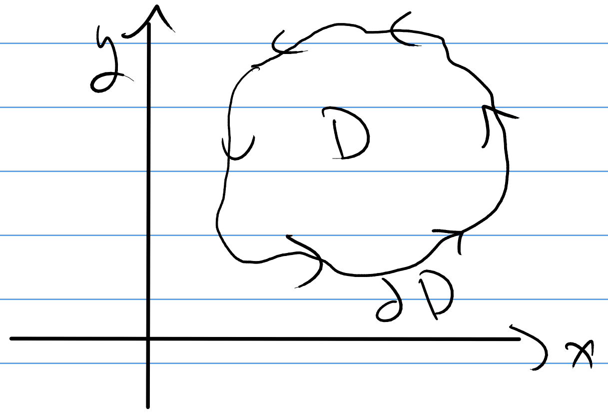
T : temp function @ (x, y, z)

Heat flow $\vec{F} = -K \nabla T$

rate of heat flow $\iint_S (-K \nabla T) \cdot d\vec{S}$

13.8

In 13.4 Green's Th^n $\mathbb{R}^2$



$$\oint_{\partial D} P\,dx + Q\,dy = \oint_{\partial D} \vec{F}\cdot d\vec{r}$$

$$F = \langle P, Q \rangle$$

Green's $$\iint_D (Q_x - P_y)\,dA = \int_{\partial D} P\,dx + Q\,dy$$

Vector form: $$\iint_D (\nabla \times \vec{F})\cdot \vec{k}\,dA = \oint_{\partial D} \vec{F}\cdot d\vec{r}$$

$\mathbb{R}^3$ Surface vs D in $\mathbb{R}^2$

S : orientated piecewise smooth

c : simple closed w/ pos. orientation

Stokes Th^n $$\boxed{\iint_S (\nabla \times \vec{F})\cdot d\vec{S} = \oint_c \vec{F}\cdot d\vec{r}}$$

$\underline{\underline{\text{Note:}}}$ $(\nabla \times \vec{F}) \cdot d\vec{S} = (\nabla \times \vec{F}) \cdot \vec{n}\, dS$

curl$(\vec{F})$

$(\vec{F} \cdot d\vec{r}) = (\vec{F} \cdot \vec{T})\, ds$

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \boxed{\oint_{\partial S} \vec{F} \cdot d\vec{r}}$$

$$= \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$\partial S \Rightarrow$ parameterize $\vec{r}(t)$

$a \leq t \leq b$

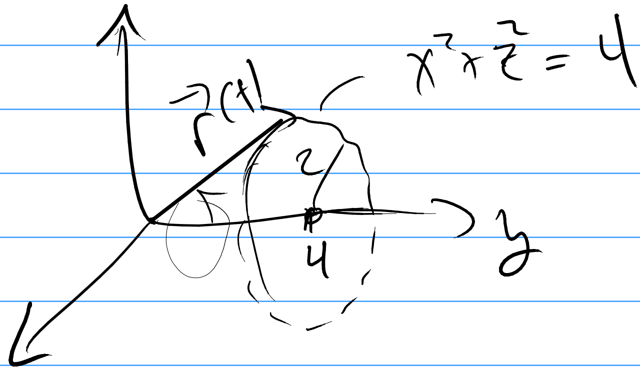
$\vec{F} = \langle x, y, xz \rangle$ curl$(\vec{F}) = \nabla \times \vec{F}$

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & xz \end{vmatrix} = \langle 0, -z, 0 \rangle$$

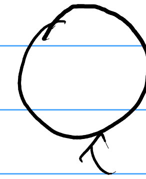
use S from start of class

$$\iint_S \langle 0, -z, 0 \rangle \cdot d\vec{S} = \int_{\partial S} \langle x, y, xz \rangle \cdot d\vec{r}$$

$$= \int_{\partial S} x\,dx + y\,dy + xz\,dz$$



∂S



$y=4$

$x^2+z^2=4$