Math 344

$\mathbb{R}^2$

Green's Vector form using div( )

$$\int_C \vec{F}\cdot\vec{n}\,ds = \iint_D \operatorname{div}(\vec{F}(x,y))\,dA$$

$\mathbb{R}^3$ (Divergence Th$^{\underline{m}}$) S is a surface in $\mathbb{R}^3$

- → E is a simple solid region
- → S is the boundary surface of E
- → S has pos (outward) orientation
- → $\vec{F}$ has cont partials in a open region containing E.

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iint_S \vec{F}\cdot d\vec{S}$$

$$= \iiint_E \underbrace{\operatorname{div}(\vec{F})}_{\text{Scalar.}}\,dV$$

why?

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \text{flux} \qquad \vec{F} = \langle P, Q, R \rangle$$

$$\Rightarrow \iiint_E (P_x + Q_y + R_z)\, dV$$

(1) A lot of tools for $\iiint_E (\ )\, dV$

(2) $P_x + Q_y + R_z$ 'may' be easier to deal with

ex

$$\iint_S \vec{F} \cdot \vec{n}\, dS$$

$$\vec{F} = \langle x^3 + y^3 + \sin(z),\ z\sin(x^3) + y^3,\ z^3 + x\cos(y^2 + x) \rangle$$

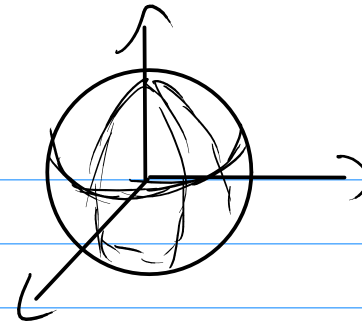
over E sphere @ origin & radius 2.

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_E (3x^2 + 3y^2 + 3z^2)\, dV$$

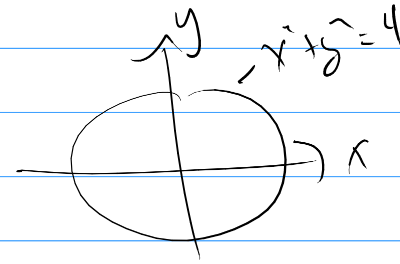
E ← sphere

ch 12 prob.

$$3 \iiint_E (x^2+y^2+z^2)\,dV$$



Cartesian: $dV = dx\,dy\,dz$



Sphere: $x^2+y^2+z^2=4$

$$3 \int_{x=-2}^{x=2} \int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} \int_{z=-\sqrt{4-x^2-y^2}}^{z=+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dy$$

Spherical

$$dV = \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$x^2+y^2+z^2 = \rho^2$$

$$= 3\int_0^{\pi}\int_0^{2\pi}\int_0^{2} (\rho^2)\,\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

$$= 3\int_0^{\pi} \sin\phi\, d\phi \int_0^{2\pi} d\theta \int_0^2 \rho^4\, d\rho$$

$$= 3\,(2)\,(2\pi)\left(\frac{1}{5}2^5\right)$$

$$= \frac{3(128)}{5}\pi = \boxed{\frac{384}{5}\pi}$$

$\boxed{\text{ex}}$ $\vec{F} = \langle x^2 \sin y, x\cos y, -xz\sin y \rangle$

$S: \quad x^8 + y^8 + z^8 = 8$

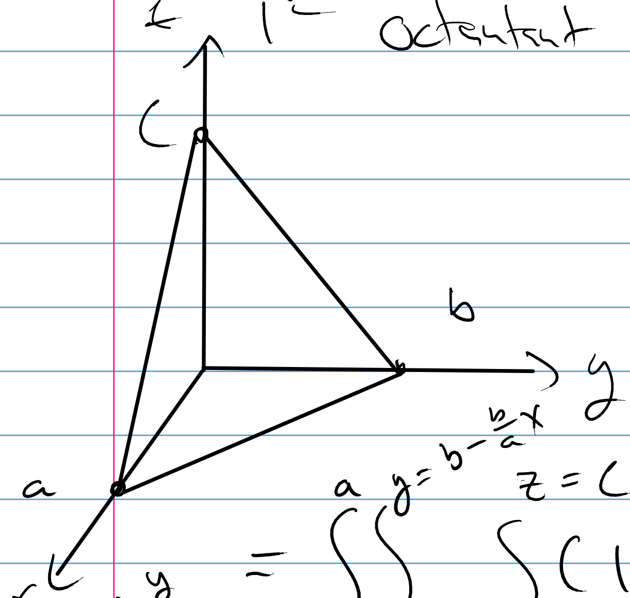
$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_E \operatorname{div}(\vec{F})\,dV$$

$$\iiint_E 2x\sin y + (-x\sin y) + (-x\sin y)\,dV$$

$$= \iiint_E (0)\,dV = 0$$

$\boxed{\text{ex}}$ $\vec{F} = \langle z, y, zx \rangle$

1^{st} octant and $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$



$$\iint_S \vec{F}\cdot\vec{n}\,dS$$

$$= \iiint_E (0 + 1 + x)\,dV$$

$$= \int_0^a \int_{y=0}^{b-\frac{b}{a}x} \int_{z=0}^{c-\frac{c}{a}x-\frac{c}{b}y} (1+x)\,dz\,dy\,dx$$