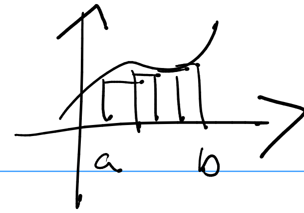
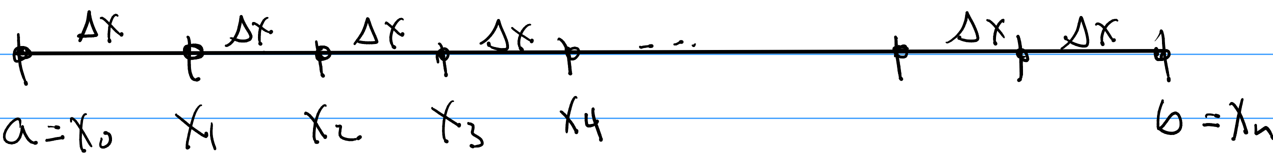
Numeric Integration

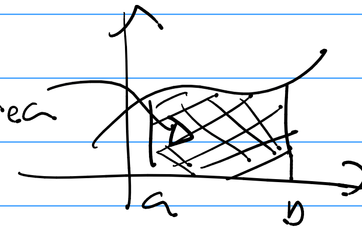


→ uniform quadrature by n-intervals

Left endpoint, Right endpoint, Trapezoidal, Simpson

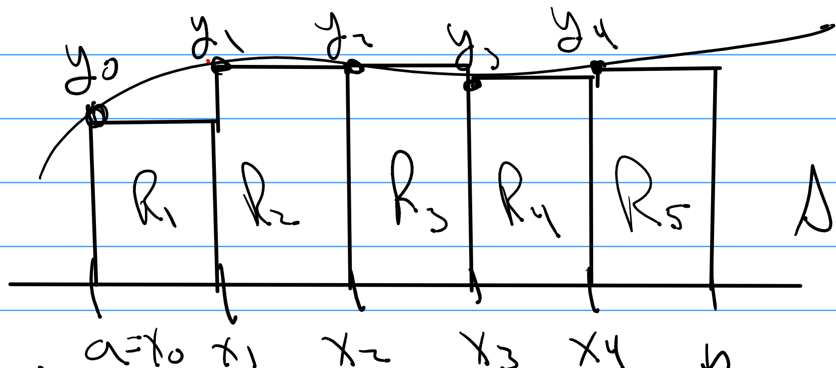


$\Delta x = \frac{b-a}{n}$ $\int_a^b f(x)\,dx = \text{Area}$



rectangle approximation ..

attach at left



$\Delta x = \frac{b-a}{5}$

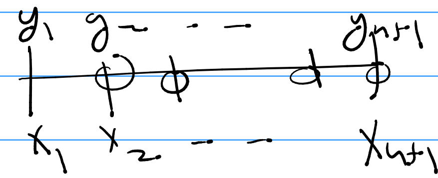
→ Left end point approx $= L_n$

Ex

$$\text{Area} \approx R_1 + R_2 + R_3 + R_4 + R_5$$
$$\Delta x\, y_0 + \Delta x\, y_1 + \cdots + \Delta x\, y_4$$
$$L_5 = \frac{b-a}{5}(y_0 + y_1 + y_2 + y_3 + y_4)$$
$$L_5 = \underline{(b-a)}\left(\frac{y_0 + y_1 + y_2 + y_3 + y_4}{5}\right)$$

% n is intervals → n+1 is points

```
x = linspace(a,b,n+1);
y = f(x);
dx = (b-a)./n;
```




```
Ln = dx.*sum(y(1:end-1));
Rn = dx.*sum(y(2:end));
```

Trap: $Trap = \frac{\Delta x}{2}\left(y_1 + 2y_2 + 2y_3 + \dots + 2y_n + y_{n+1}\right)$

Simp: (n must be even)

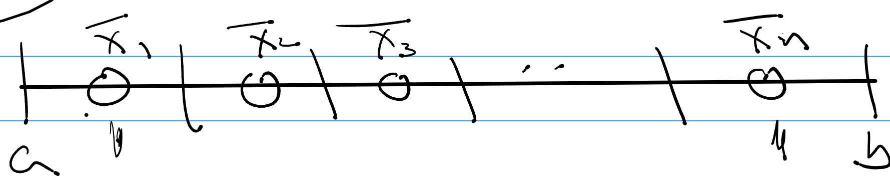
$$Simp = \frac{\Delta x}{3}\left(y_1 + 4y_2 + 2y_3 + 4y_4 + \dots + 2y_{n-1} + 4y_n + y_{n+1}\right)$$

```
y(2:end-1) = 2.*y(2:end-1);
Tn = (dx./2).*sum(y);
```

or

```
y(2:2:end-1) = 4.*y(2:2:end-1);
y(3:2:end-2) = 2.*y(3:2:end-2);
Sn = (dx./3).*sum(y);
```

Midpoint




```
dx = (b-a)./n;
x = linspace(a+dx/2, b-dx/2, n);
y = f(x)
Mn = dx.*sum(y);
```

$$\int_0^1 x^3\,dx = \frac{1}{4}x^4\Big|_0^1 = 1/4$$