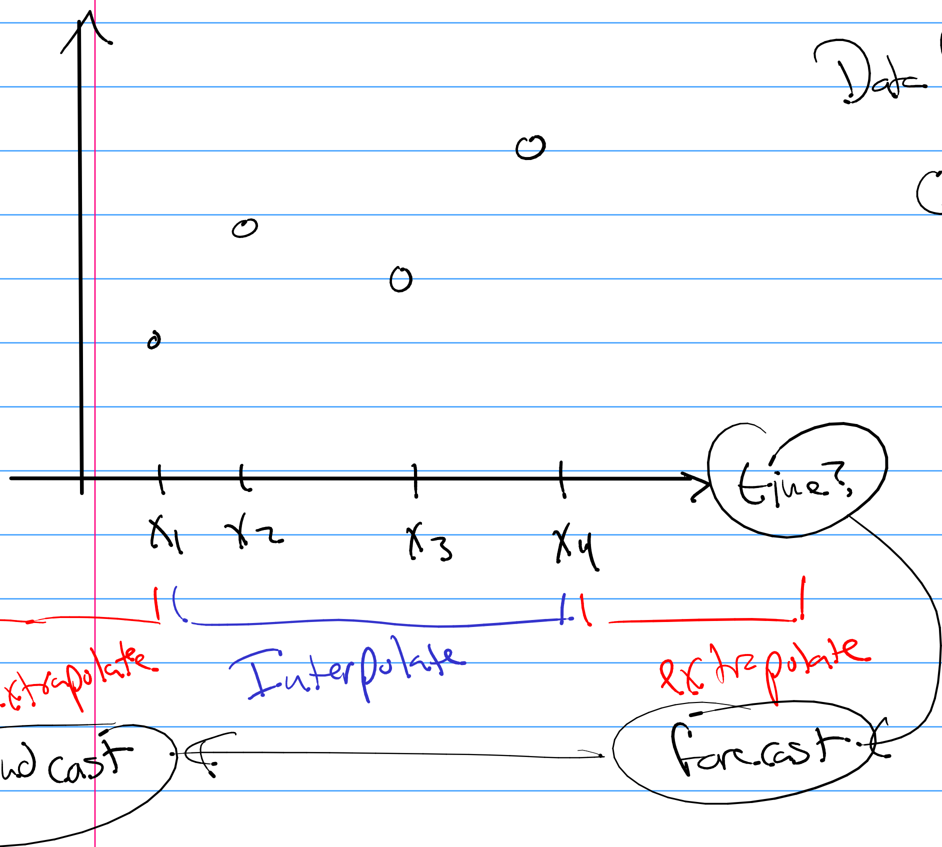
Math 451

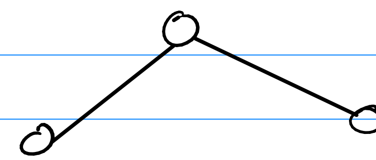
Data fitting



① K points single soln & K term poly.

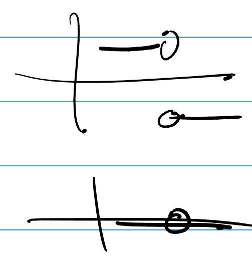
$p(x) \leftarrow$ interpolating polynomial

②

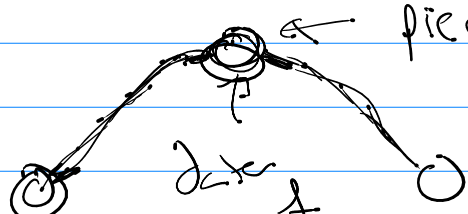


linear piecewise function

f = position
f' = velocity
f'' = accel



③



data known: f
f' ?
f'' ?

← piecewise curve that
a) Same position
b) Same slope
b) Same curvature

(See text)

Interpolating Polynomial

k data points $\rightarrow p_1, p_2, \ldots, p_k$

Form: system of linear eqn's

$x \equiv x$-coord, $y \equiv y$-coord } data

$$\underbrace{[x.^{\wedge}k\text{-}1 \;\; \cdots \;\; x.^{\wedge}1 \;\; x.^{\wedge}0]}_{\text{Vandermonde matrix of } x} \, c = y$$

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to get data $\rightarrow$ ginput()

ex [xd yd] = ginput(7)

$$7 \text{ points} \rightarrow p(x) = \underline{a}x^6 + \underline{b}x^5 + \underline{c}\,x^4 + \underline{d}\,x^3 + \underline{e}x^2 + \underline{f}x + \underline{g}$$

Solve $\boxed{Vc = y}$ (See video)

Interpolating Polynomial

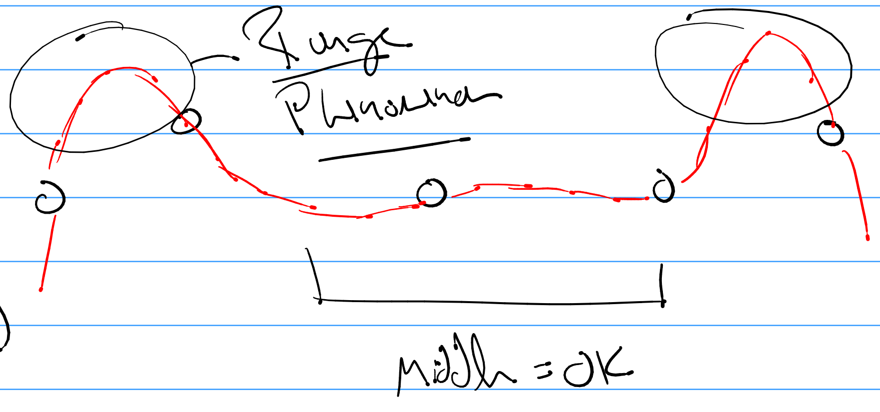
Lagrange form:

(x_1, y_1) (x_2, y_2) (x_3, y_3)

$$p(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}y_3$$

n-points: $$P(x) = \sum_{k=1}^{n} \left(\prod_{j\neq k} \frac{(x-x_j)}{(x_k-x_j)} \right) y_k$$

Notice



(See video)

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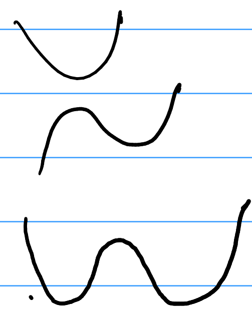
ex: 30 data points → $P(x)$ is deg 29 → 28 bends

ex:
deg 2 x^2
deg 3 x^3
deg 4 x^4



→ poly interp. of large degree (lots of data) are "bad"

→ fixes? → "better" poly's of deg $(k-1)$ to pick?

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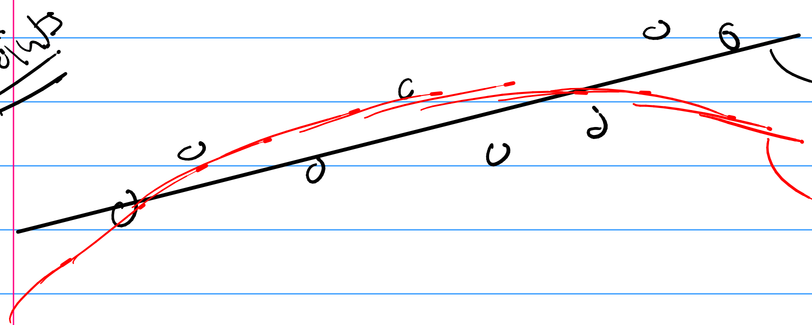
leave interpolating poly → don't hit the points.

get close.

$\boxed{\text{approx on the data}}$

Interp. Poly = deg 7

8 points



linear approx. = deg 1

quad. approx = deg 2

etc

Problem: 3 data points P_1, P_2, P_3

linear approx: $l(x) = c_1 x + c_2$

P_1 : $c_1 x_1 + c_2 = y_1$
P_2 : $c_1 x_2 + c_2 = y_2$
P_3 : $c_1 x_3 + c_2 = y_3$

$\xrightarrow[\text{matrix}]{\text{as}}$ $\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

overdetermined!

→ have no soln.

by Linear Algebra. overdet $Ax = b$

Solve $\boxed{A^T A x = A^T b}$ has a soln

→ least squares soln.