

Math 451

Calc Toolbox

- ① Trig
- ② Derivatives
- ③ Def. Integrals
- ④ Data Fitting
- ⑤ Poly. Interpolation

Next Proj.

Next
Next
Proj.

Adaptive Quad.

```
function a = recsimp(f,a,b)
```

```
x = linspace(a,b,5);
```

```
y = f(x);
```

```
S2 = (b-a)*(y(1) + 4.*y(3) + y(5))./6;
```

```
S4 = (b-a)*(y(1) + 4.*y(2) + 2.*y(3) + 4.*y(4) + y(5))./12;
```

```
if abs(S4 - S2) < 1e-10
```

```
    a = S4;
```

```
else
```

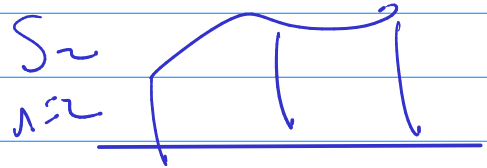
```
    leftside = recsimp(f,a,x(3));
```

```
    rightside = recsimp(f,x(3),b);
```

```
    a = leftside + rightside;
```

```
end
```

```
end
```

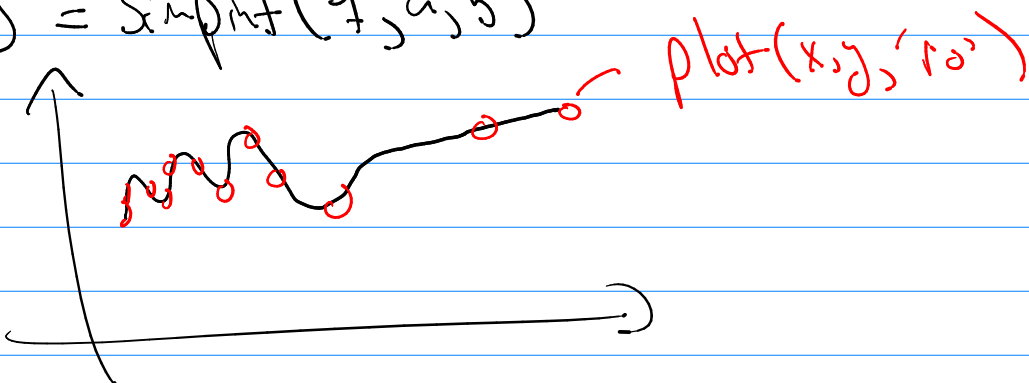


two approx

Improve?

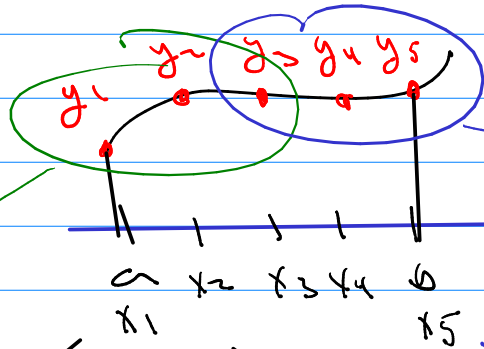
① keep x's and y's of algorithm?

```
[x,y] = simpint(f,a,b)
```

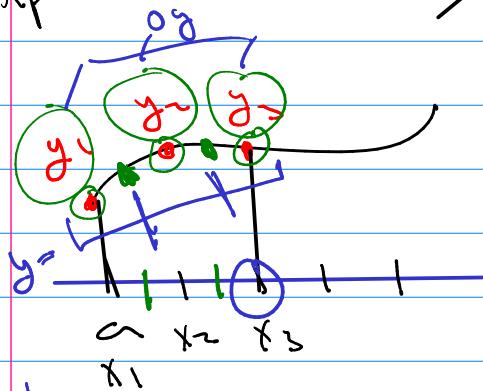


② Performance?

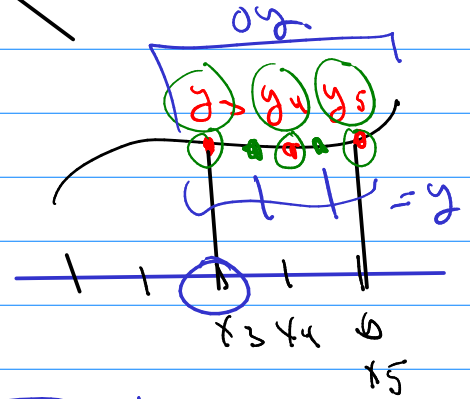
$\text{Simpint}(f, a, b)$



$\text{Simpint}(f, a, x_3)$



$\text{Simpint}(f, x_3, b)$



$x = \text{ linspace}(a, x_3, 5)$

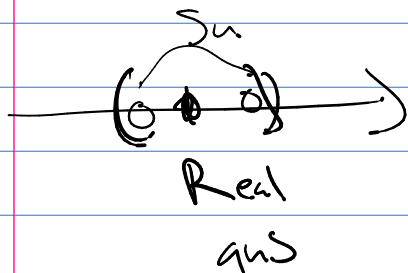
$y = f(x) \rightarrow 5$ function calls
etc
but 3 were known

$x = \text{ linspace}(x_3, b, 5)$

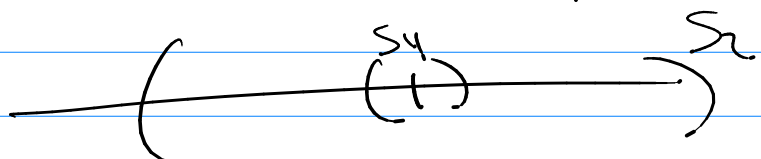
$y = f(x)$
etc

$\rightarrow 10$ function calls when 4 are really needed.

$$|E_S| \leq \frac{K(b-a)^5}{18n^4} \rightarrow |E_S| \propto \frac{1}{n^4}$$



S_u -Rule $\propto \frac{1}{n^4}$



$$S_2 (n=2) \xrightarrow{\times 2} S_4 (n=4)$$

$$\text{Error of } S_4 = \frac{1}{2^4} \text{Error of } S_2$$

$$\text{Error of } S_2 = 16 \text{Error of } S_4$$

$$S_2 - \tilde{R} = 16(S_4 - \hat{R})$$

$$S_2 - \hat{R} = 16S_4 - 16\hat{R}$$

$$15\hat{R} = 16S_4 - S_2$$

$$15\hat{R} = 15S_4 + (S_4 - S_2)$$

$$\boxed{\hat{R} = S_4 + \frac{1}{15}(S_4 - S_2)}$$